Advanced Complexity

TD n°4

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Exercise 1 : Language theory

Show that the following problems are PSPACE-complete :

1. NFA Universality :
   — INPUT : a non-deterministic automaton A over alphabet Σ
   — QUESTION : $\mathcal{L}(A) = \Sigma^*$?
   Bonus : what is the complexity of this problem for a DFA ?
2. NFA Equivalence
   — INPUT : two non-deterministic automata A_1 and A_2 over the same alphabet Σ
   — QUESTION : $L(A_1) = L(A_2)$
   Bonus : what is the complexity of this problem for a DFA ?
3. DFA Intersection Vacuity :
   — INPUT : deterministic automata $A_1, \ldots, A_m$ for some m
   — QUESTION : $\bigcap_{i=1}^{m} L(A_i) = \emptyset$?

Exercise 2 : Did you get padding ?

Show that if $\mathsf{P} = \mathsf{PSPACE}$, then $\mathsf{EXPTIME} = \mathsf{EXPSPACE}$.

Exercise 3 : Too fast !

Show that $\mathsf{ATIME}(log\ n) \neq L$.

Exercise 4 : Direct application

Show that $\mathsf{EXPSPACE} = \mathsf{AEXPTIME}$.
Hint : You may use that if f is space-constructible, then :

$$\mathsf{SPACE}(poly(f(n)) = \mathsf{ATIME}(poly(f(n)))$$

Exercise 5 : Closure under morphisms

Given a finite alphabet Σ, a function $f : \Sigma^* \to \Sigma^*$ is a morphism if $f(\Sigma) \subseteq \Sigma$ and for all $a = a_1 \cdots a_n \in \Sigma^*$, $f(a) = f(a_1) \cdots f(a_n)$ (f is uniquely determined by the value it takes on Σ).

1. Show that NP is closed under morphisms, that is : for any language $L \in \mathsf{NP}$, and any morphism f on the alphabet of L, $f(L) \in \mathsf{NP}$.
2. Show that if P is closed under morphisms, then $\mathsf{P} = \mathsf{NP}$.

Exercise 6 : Padding and unary languages 1.

Recall that a *unary* language is any language over a one-letter alphabet. Prove that if a unary language is NP-complete, then $\mathsf{P} = \mathsf{NP}$.

2. Prove that if every unary language in NP is actually in P, then $\mathsf{EXP} = \mathsf{NEXP}$.
3. Prove that if $\mathsf{SPACE}(2^{O(n)}) = \mathsf{TIME}(2^{O(n)})$, then every unary language in PSPACE is actually in P.