Continuous Probabilistic Petri nets : specification and analysis

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From Petri nets to continuous Petri nets. Continuous Petri nets (CPN) were introduced [6] by considering continuous states (specified by a *real* number of tokens in the places) where the system's dynamics are driven (1) either by discrete events (i.e., the *partial* firing of a transition) (2) or by continuous evolution (i.e., the infinitesimal and simultaneous firing of transitions according to their rate). In the first case, these nets are called autonomous CPNs or simply CPNs, while in the second case, they are referred to as timed CPNs.

Modeling with CPNs. CPNs have been used in several significant application fields. In [4], a method based on CPNs is proposed for fault diagnosis in manufacturing systems, where such diagnosis would be impractical using traditional Petri nets (for this type of modeling, see also [15]). In [13], the authors introduce a bottom-up modeling methodology to represent cell metabolism and solve the regulation control problem within this framework. By combining Petri nets and CPNs, hybrid Petri nets are defined with applications to the modeling and simulation of water distribution systems [8] and the analysis of traffic in urban transport networks [14].

Analysis of CPNs. Although several analysis methods have been developed for timed CPNs, a general and automatic method seems out of reach, as standard problems in dynamic systems are undecidable, even in the case of bounded nets [11].

Due to the semantics of (autonomous) CPNs, a marking can be the limit of markings visited by an infinite sequence of firings. Thus, most of the usual properties are duplicated depending on whether these markings are considered or not. When considering these markings, reachability (respectively liveness, pseudo-liveness) becomes limit-reachability (respectively limit-liveness, limit-pseudo-liveness).

Unlike the timed case, the analysis of CPNs has proven to be significantly less complex than that of Petri nets. Numerous studies have shown that standard decision problems either belong to the P class or are NP-complete [10,12,11,7]. Furthermore, CPNs have facilitated the development of efficient verification tools for Petri nets [2], and recently, the interest in transfinite sequences (i.e., ordinals potentially greater than ω) in trajectory analysis has been highlighted [9].

Internship objectives. Most discrete event models with an infinite number of states have been "probabilized" (e.g., pushdown automata [3], Petri nets [5,1]) to go beyond the traditional queueing network model, which is used in performance evaluation but is unsuitable, for example, for the quantitative analysis of concurrent systems due to the lack of a realistic synchronization mechanism.

Thus, the first step will be to propose (at least) two probabilistic semantics for CPNs based on a random choice of a transition to fire:

- one based on a probabilistic distribution of the amount of firing for the chosen transition;
- the other based on the maximal firing of the chosen transition.

We will then study, within this framework, both standard problems such as the (exact or approximate) calculation of reachability probability, and specific problems such as the almost sure reachability of a limit marking (i.e., lim-accessible but unreachable).

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Location

The Master will be supervised at the
Laboratoire Méthodes Formelles
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Qualifications and Connections

This internship and the PhD are opened to strongly motivated and excellent Master students who like computer science and mathematics. Knowledges in probabilities and Petri nets models will be a plus.

Ideally, the candidate holds a Master degree in Computer Science (or Applied Mathematics) equivalently is graduated from a Computer Science (or Applied Mathematics) Engineering School. Ideally, the candidate has strong knowledge both in computer science or applied mathematics (probability, statistics). The internship is an ideal opportunity for starting a PhD thesis (that could be made after the internship).

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