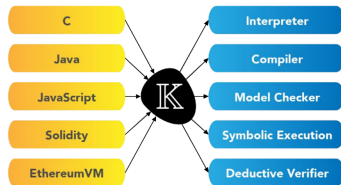


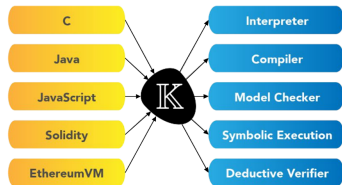
K framework in a nutshell

- Semantical framework
 - to define formal semantics of programming languages
 - to automatically generate tools from these semantics



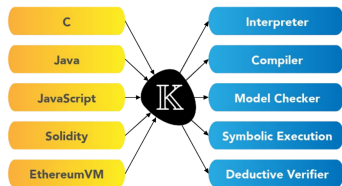
K framework in a nutshell

- Semantical framework
 - to define formal semantics of programming languages
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- Based on MATCHING LOGIC
 - an untyped 1st order logic with fixpoints and a "next" operator



\mathbb{K} framework in a nutshell

- Semantical framework
 - to define formal semantics of programming languages
 - to automatically generate tools from these semantics
- Based on MATCHING LOGIC
 - an untyped 1st order logic with fixpoints and a "next" operator



- Common feature: \mathbb{K} and DEDUKTI are based on rewriting.

Characteristic of rewriting	\mathbb{K}	DEDUKTI
At any position	✓	✓
Non-linearity	✓	✓
Conditional	✓	✗
Rewriting modulo ACUI	✓	✗

Define a semantics with \mathbb{K}

Two steps to define a \mathbb{K} semantics:

- **Syntax**
- **Semantics**

Define a semantics with \mathbb{K}

Two steps to define a \mathbb{K} semantics:

- **Syntax**
 - **BNF grammar**
- **Semantics**

Define a semantics with \mathbb{K}

Two steps to define a \mathbb{K} semantics:

- **Syntax**
 - **BNF grammar**
- **Semantics**
 - **Configuration** = State of the program
Example: $\langle \langle x + 17 \rangle_k \langle x \mapsto 25 \rangle_{env} \rangle$
 - **Rewriting rule** on configurations (\sim transition system)

Define a semantics with \mathbb{K}

$\langle x = 1 ; \text{while } 0 < x \{ x-- \} ; \rangle_k$
 $\langle \text{nil} \rangle_{env}$

$\langle \text{while } 0 < x \{ x-- \} ; \rangle_k$
 $\langle x \mapsto 1 \rangle_{env}$

$\langle \text{while } 0 < x \{ x-- \} ; \rangle_k$
 $\langle x \mapsto 42 \rangle_{env}$

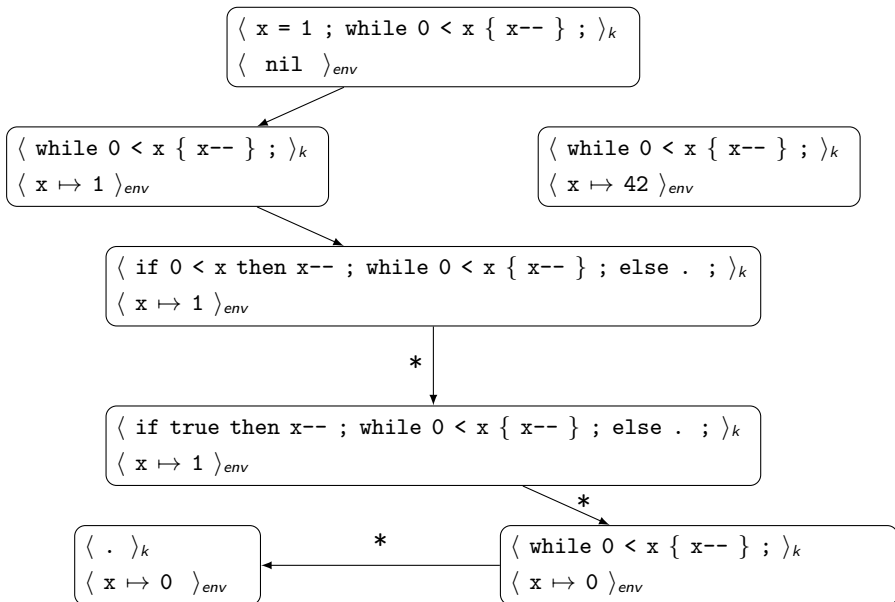
$\langle \text{if } 0 < x \text{ then } x-- ; \text{while } 0 < x \{ x-- \} ; \text{else } . ; \rangle_k$
 $\langle x \mapsto 1 \rangle_{env}$

$\langle \text{if true then } x-- ; \text{while } 0 < x \{ x-- \} ; \text{else } . ; \rangle_k$
 $\langle x \mapsto 1 \rangle_{env}$

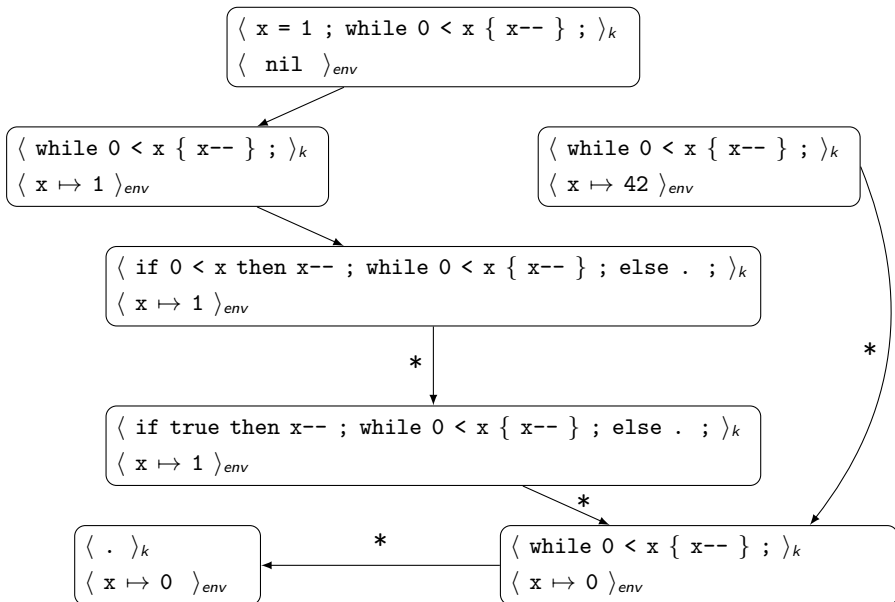
$\langle . \rangle_k$
 $\langle x \mapsto 0 \rangle_{env}$

$\langle \text{while } 0 < x \{ x-- \} ; \rangle_k$
 $\langle x \mapsto 0 \rangle_{env}$

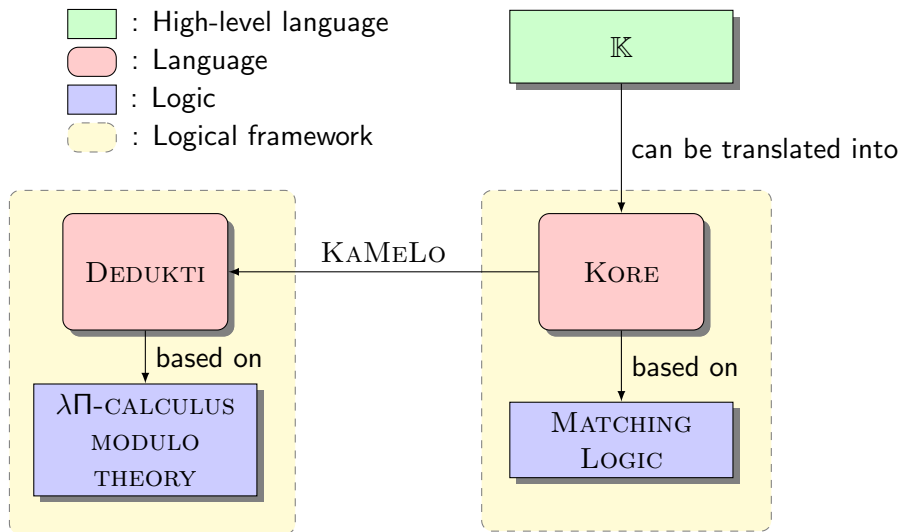
Define a semantics with \mathbb{K}



Define a semantics with \mathbb{K}



Pipeline of the translation



How to do it?

~~How to do it?~~

- **What is the purpose of the translation?**

~~How to do it?~~

- **What is the purpose of the translation?**
- **What do we want to do with the result of the translation?**
 - Execute a program? \rightsquigarrow shallow encoding
 - Check a proof? \rightsquigarrow deep encoding

① A shallow encoding to execute a program in DEDUKTI

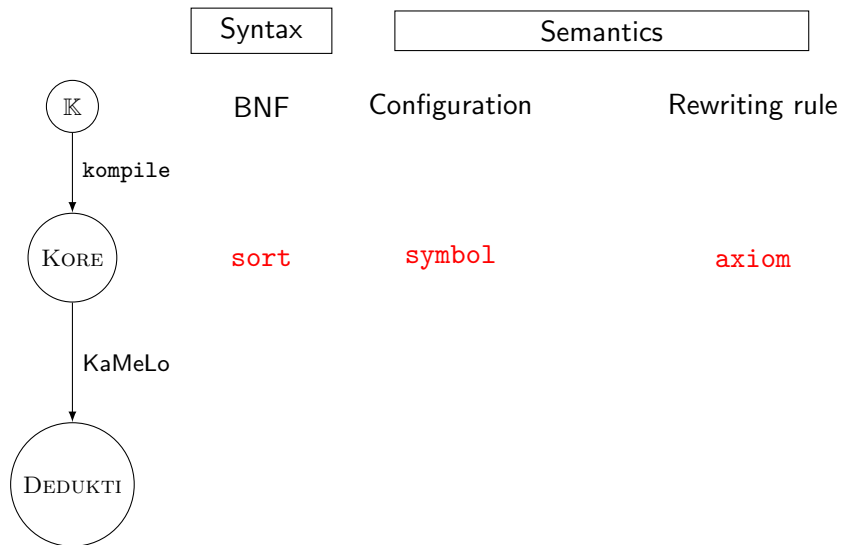
② A deep encoding to check proofs in DEDUKTI

Translate MATCHING LOGIC constructors, notations and symbols

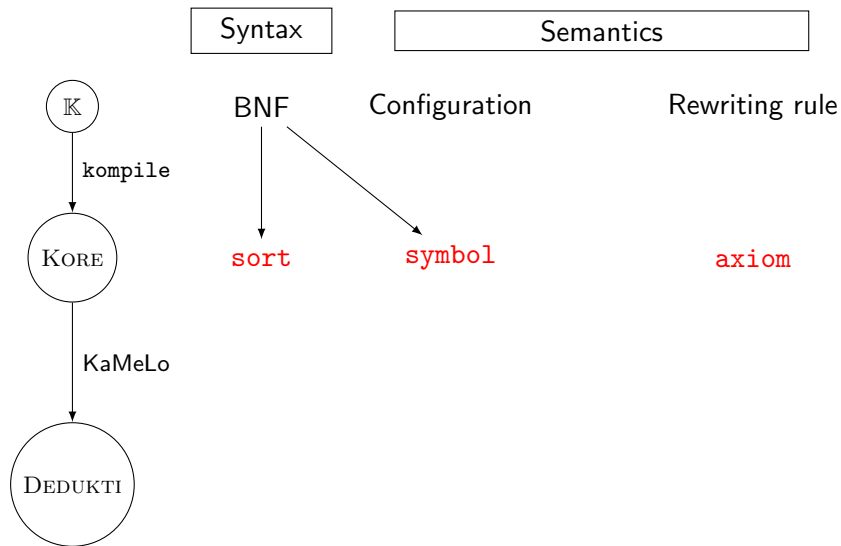
Translate MATCHING LOGIC proof system

③ Conclusion

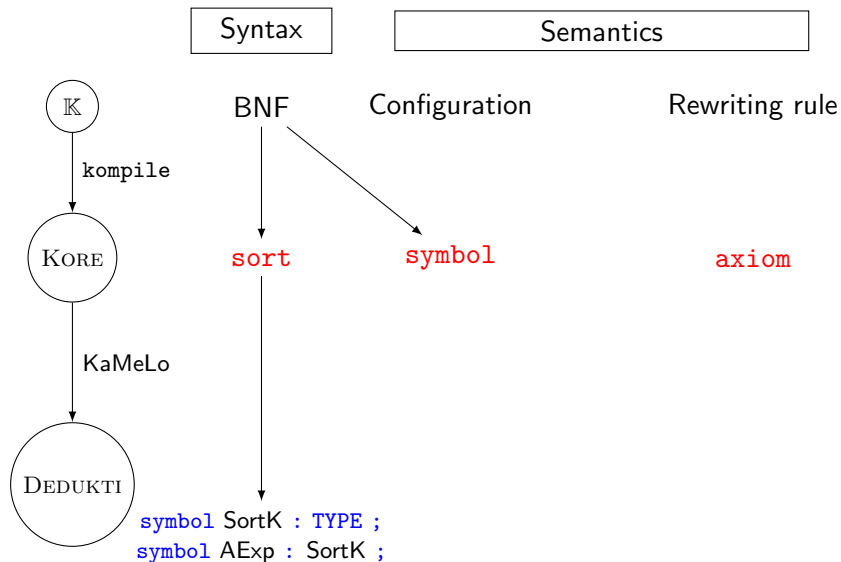
Translation from \mathbb{K} to DEDUKTI



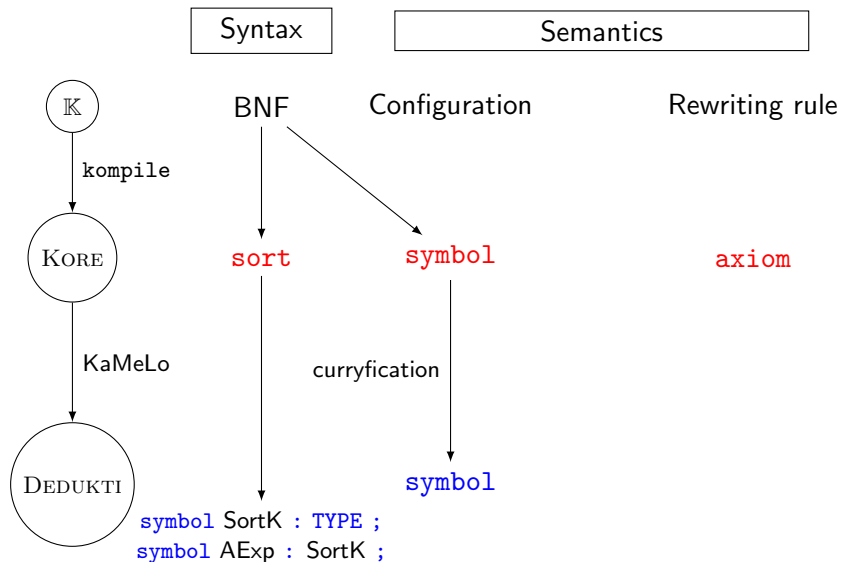
Translation from \mathbb{K} to DEDUKTI



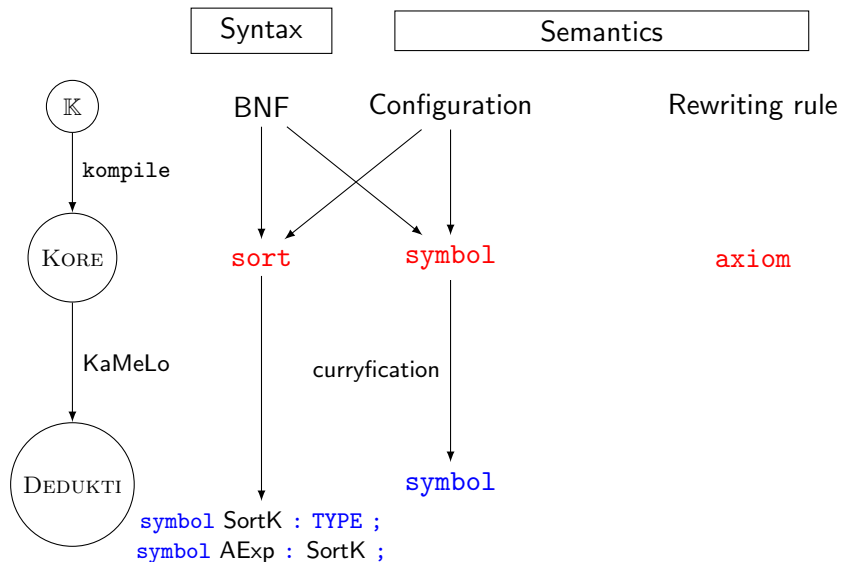
Translation from \mathbb{K} to DEDUKTI



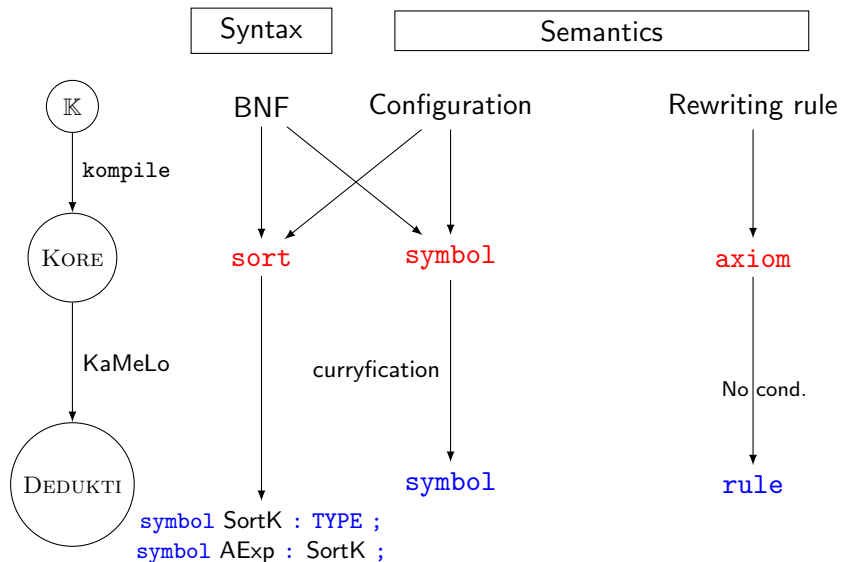
Translation from \mathbb{K} to DEDUKTI



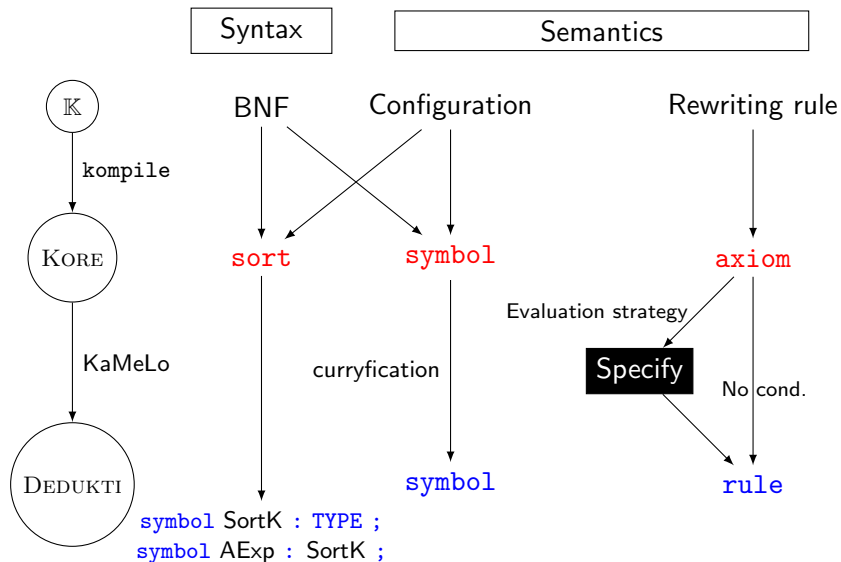
Translation from \mathbb{K} to DEDUKTI



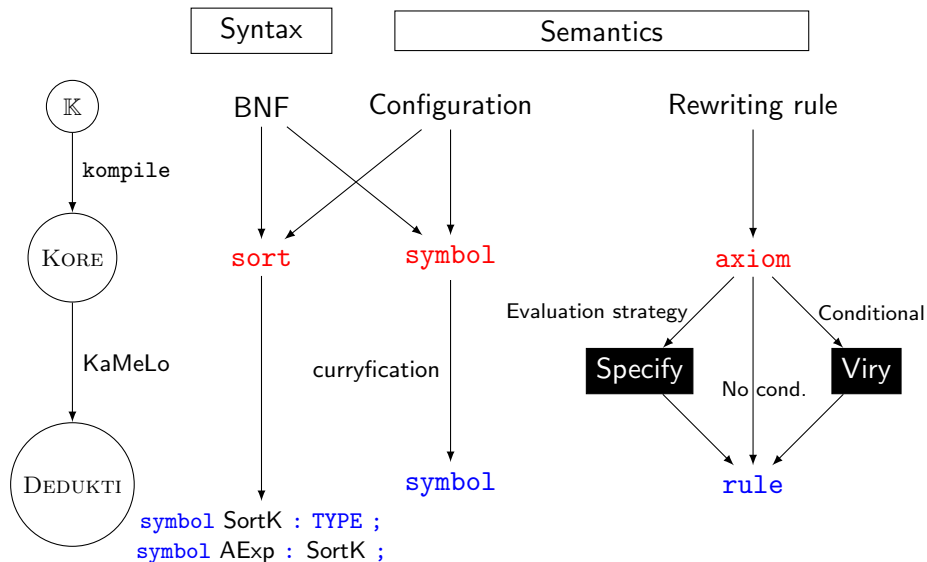
Translation from \mathbb{K} to DEDUKTI



Translation from \mathbb{K} to DEDUKTI



Translation from \mathbb{K} to DEDUKTI



Translate evaluation strategies

Generated rules to define evaluation strategies:

Key ideas:

- **Evaluation is ordering thanks to a list.**
- **Evaluated expressions have a specific type.**

Translate evaluation strategies

Generated rules to define evaluation strategies:

1. rule $E_1 \text{ and } E_2 \Rightarrow E_1 \curvearrowright (*_{\text{and}}^1 E_2)$ requires $E_1 \notin \text{Bool}$
2. rule $E_1 \curvearrowright (*_{\text{and}}^1 E_2) \Rightarrow E_1 \text{ and } E_2$ requires $E_1 \in \text{Bool}$

Translation into DEDUKTI:

1. Instantiation of E_1 :

- a. rule $\langle (\text{not } \$X1) \text{ and } \$E2 \curvearrowright \$s \rangle_k$
 $\hookrightarrow \langle (\text{not } \$X1) \curvearrowright (*_{\text{and}}^1 \$E2) \curvearrowright \$s \rangle_k$
 - b. rule $\langle (\$X1 \text{ and } \$X2) \text{ and } \$E2 \curvearrowright \$s \rangle_k$
 $\hookrightarrow \langle (\$X1 \text{ and } \$X2) \curvearrowright (*_{\text{and}}^1 \$E2) \curvearrowright \$s \rangle_k$
2. rule $\langle (\text{inj } \$E1) \curvearrowright (*_{\text{and}}^1 \$E2) \curvearrowright \$s \rangle_k$
 $\hookrightarrow \langle (\text{inj } \$E1) \text{ and } \$E2 \curvearrowright \$s \rangle_k$

The grammar of
BExp:

```
syntax BExp ::= Bool
              | "not" BExp
              > BExp "and" BExp
              | "(" BExp ")"
```

Translate a CTRS to a TRS¹

¹Patrick Viry, *Elimination of Conditions*, Journal of Symbolic Computation, 1999

Translate a CTRS to a TRS¹

- **Example 1:**

(1) `rule max X Y => Y requires X <Int Y`

(2) `rule max X Y => X requires X >=Int Y`

¹Patrick Viry, *Elimination of Conditions*, Journal of Symbolic Computation, 1999

Translate a CTRS to a TRS¹

- **Example 1:**

(1) **rule** $max\ X\ Y \Rightarrow Y$ **requires** $X <Int\ Y$

(2) **rule** $max\ X\ Y \Rightarrow X$ **requires** $X \geq Int\ Y$

translated into

(0) **rule** $max\ \$x\ \$y \leftrightarrow bmax\ \$x\ \$y\ (\$x < \$y)\ (\$x \geq \$y)$

¹Patrick Viry, *Elimination of Conditions*, Journal of Symbolic Computation, 1999

Translate a CTRS to a TRS¹

- **Example 1:**

(1) **rule** $max\ X\ Y \Rightarrow Y$ **requires** $X <Int\ Y$

(2) **rule** $max\ X\ Y \Rightarrow X$ **requires** $X \geq Int\ Y$

translated into

(0) **rule** $max\ \$x\ \$y \hookrightarrow bmax\ \$x\ \$y\ (\$x < \$y)\ (\$x \geq \$y)$

(1') **rule** $bmax\ \$x\ \$y\ true\ _ \hookrightarrow \y

(2') **rule** $bmax\ \$x\ \$y\ _ \ true \hookrightarrow \x

¹Patrick Viry, *Elimination of Conditions*, Journal of Symbolic Computation, 1999

Translate a CTRS to a TRS¹

- **Example 1:**

(1) rule $\text{max } X \ Y \Rightarrow Y$ requires $X <_{\text{Int}} Y$

(2) rule $\text{max } X \ Y \Rightarrow X$ requires $X \geq_{\text{Int}} Y$

translated into

(0) rule $\text{max } \$x \ \$y \hookrightarrow \text{bmax } \$x \ \$y \ (\$x < \$y) \ (\$x \geq \$y)$

(1') rule $\text{bmax } \$x \ \$y \ \text{true} \ _ \hookrightarrow \y

(2') rule $\text{bmax } \$x \ \$y \ _ \ \text{true} \hookrightarrow \x

- **Example 2:**

(A) rule $\text{max } X \ Y \Rightarrow Y$ requires $X <_{\text{Int}} Y$

(B) rule $\text{max } X \ Y \Rightarrow X$ [otherwise]

translated into

(N) rule $\text{max } \$x \ \$y \hookrightarrow \text{bmax } \$x \ \$y \ (\$x < \$y)$

(A') rule $\text{bmax } \$x \ \$y \ \text{true} \hookrightarrow \y

(B') rule $\text{bmax } \$x \ \$y \ \text{false} \hookrightarrow \x

¹Patrick Viry, *Elimination of Conditions*, Journal of Symbolic Computation, 1999

① A shallow encoding to execute a program in DEDUKTI

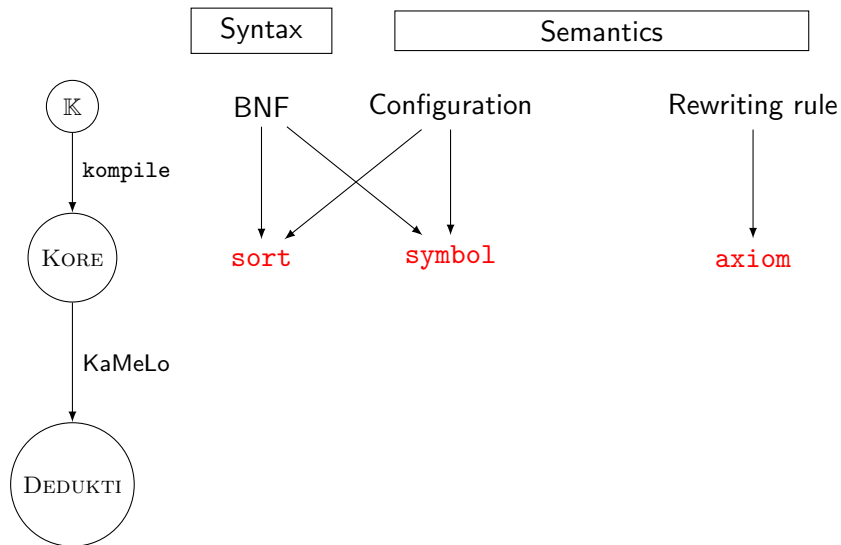
② A deep encoding to check proofs in DEDUKTI

Translate MATCHING LOGIC constructors, notations and symbols

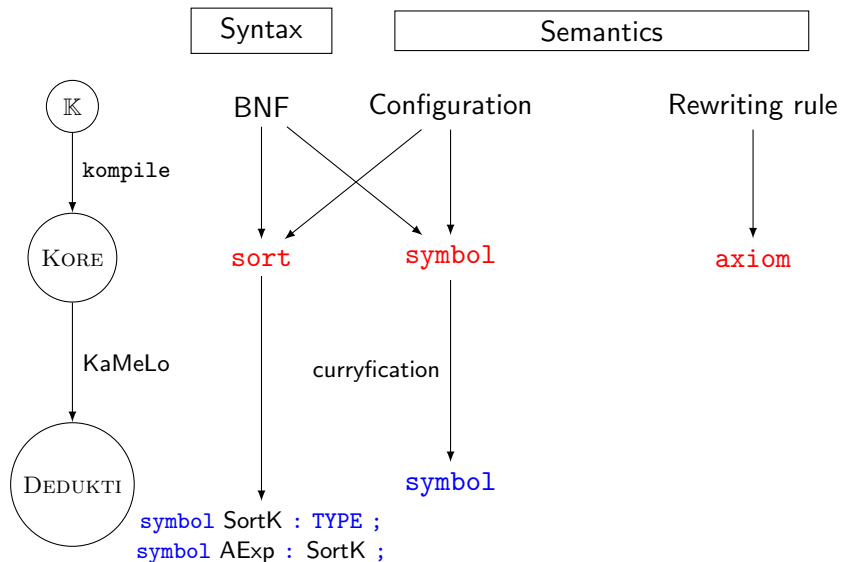
Translate MATCHING LOGIC proof system

③ Conclusion

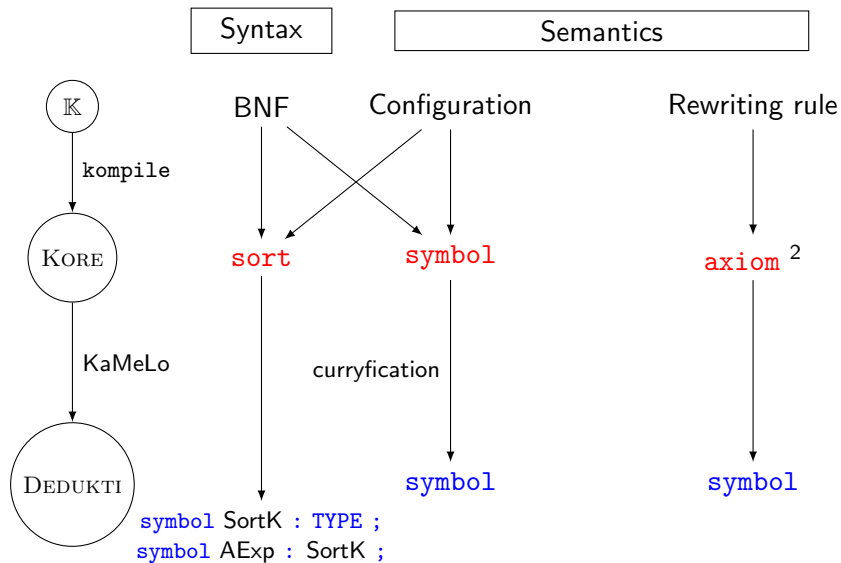
Translation from \mathbb{K} to DEDUKTI



Translation from \mathbb{K} to DEDUKTI



Translation from \mathbb{K} to DEDUKTI



²MATCHING LOGIC pattern

① A shallow encoding to execute a program in DEDUKTI

② A deep encoding to check proofs in DEDUKTI

Translate MATCHING LOGIC constructors, notations and symbols

Translate MATCHING LOGIC proof system

③ Conclusion

MATCHING LOGIC constructors

MATCHING LOGIC defines patterns

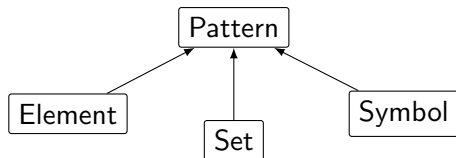
$\varphi ::= x \mid X \mid \sigma \mid \varphi @ \varphi \mid \perp \mid \varphi \rightarrow \varphi \mid \exists x.\varphi \mid \mu X.\varphi$

MATCHING LOGIC constructors

MATCHING LOGIC defines patterns

$\varphi ::= x \mid X \mid \sigma \mid \varphi @ \varphi \mid \perp \mid \varphi \rightarrow \varphi \mid \exists x.\varphi \mid \mu X.\varphi$

```
symbol #Pattern    : TYPE;  
symbol #Element    : TYPE;  
symbol #Set        : TYPE;  
symbol #Symbol     : TYPE;
```



The next symbol:

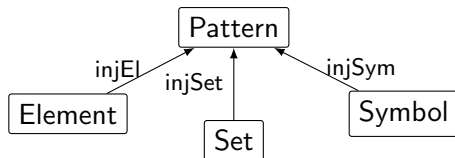
```
symbol • : #Symbol; // Symbol
```

MATCHING LOGIC constructors

MATCHING LOGIC defines patterns

$\varphi ::= x \mid X \mid \sigma \mid \varphi @ \varphi \mid \perp \mid \varphi \rightarrow \varphi \mid \exists x.\varphi \mid \mu X.\varphi$

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symbol #Pattern    : TYPE;  
symbol #Element    : TYPE;  
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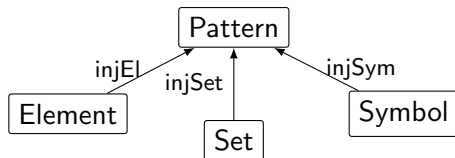
```
symbol injEl      : #Element → #Pattern;  
symbol injSet     : #Set → #Pattern;  
symbol injSym     : #Symbol → #Pattern;
```

MATCHING LOGIC constructors

MATCHING LOGIC defines patterns

$\varphi ::= x \mid X \mid \sigma \mid \varphi @ \varphi \mid \perp \mid \varphi \rightarrow \varphi \mid \exists x.\varphi \mid \mu X.\varphi$

```
symbol #Pattern    : TYPE;  
symbol #Element    : TYPE;  
symbol #Set        : TYPE;  
symbol #Symbol     : TYPE;
```



```
symbol injEl      : #Element → #Pattern;  
symbol injSet     : #Set → #Pattern;  
symbol injSym     : #Symbol → #Pattern;
```

```
symbol @ML      : #Pattern → #Pattern → #Pattern;  
symbol ⊥ML     : #Pattern;  
symbol ⇒ML     : #Pattern → #Pattern → #Pattern;  
symbol ∃ML     : (#Element → #Pattern) → #Pattern;  
symbol μML     : (#Set → #Pattern) → #Pattern;
```


Notations vs Symbols

- Notations are syntactic sugar:

```
symbol  $\neg_{ML} : \#Pattern \rightarrow \#Pattern;$   
rule  $\neg_{ML} \ \$\varphi \hookrightarrow \ \$\varphi \Rightarrow_{ML} \perp_{ML};$ 
```

```
symbol  $\vee_{ML} : \#Pattern \rightarrow \#Pattern \rightarrow \#Pattern;$   
rule  $\ \$\varphi_0 \vee_{ML} \ \$\varphi_1 \hookrightarrow (\neg_{ML} \ \$\varphi_0) \Rightarrow_{ML} \ \$\varphi_1;$ 
```

- Symbols are patterns:

```
symbol  $\bullet : \#Symbol; // \textit{Symbol}$   
symbol  $\rightsquigarrow : \#Pattern \rightarrow \#Pattern \rightarrow \#Pattern; // \textit{Notation}$   
rule  $\ \$\varphi_1 \rightsquigarrow \ \$\varphi_2 \hookrightarrow \ \$\varphi_1 \Rightarrow_{ML} ((injSym \ \bullet) @_{ML} \ \$\varphi_2);$ 
```

① A shallow encoding to execute a program in DEDUKTI

② A deep encoding to check proofs in DEDUKTI

Translate MATCHING LOGIC constructors, notations and symbols

Translate MATCHING LOGIC proof system

③ Conclusion

MATCHING LOGIC proof system

FOL Reasoning

$$\frac{}{\varphi \rightarrow (\psi \rightarrow \varphi)} \text{ (Prop 1)}$$

$$\frac{}{(\varphi \rightarrow (\psi \rightarrow \theta)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \theta))} \text{ (Prop 2)}$$

$$\frac{}{((\varphi \rightarrow \perp) \rightarrow \perp) \rightarrow \varphi} \text{ (Prop 3)}$$

$$\frac{\varphi_1 \quad \varphi_1 \rightarrow \varphi_2}{\varphi_2} \text{ (Modus Ponens)}$$

$$\frac{}{\varphi[y/x] \rightarrow \exists x.\varphi} \text{ (\exists-Quantifier)}$$

$$\frac{\varphi_1 \rightarrow \varphi_2 \quad (\text{when } x \notin FV(\varphi_2))}{(\exists x.\varphi_1) \rightarrow \varphi_2} \text{ (\exists-Generalization)}$$

Technical rules

$$\frac{}{\exists x.x} \text{ (Existence)}$$

$$\frac{}{\neg(C_1[x \wedge \varphi] \wedge C_2[x \wedge \neg\varphi])} \text{ (Singleton)}$$

Frame Reasoning

$$\frac{}{C[\perp] \rightarrow \perp} \text{ (Propagation}_{\perp})$$

$$\frac{}{C[\varphi_1 \vee \varphi_2] \rightarrow C[\varphi_1] \vee C[\varphi_2]} \text{ (Propagation}_{\vee})$$

$$\frac{(\text{when } x \notin FV(C))}{C[\exists x.\varphi] \rightarrow \exists x.C[\varphi]} \text{ (Propagation}_{\exists})$$

$$\frac{\varphi_1 \rightarrow \varphi_2}{C[\varphi_1] \rightarrow C[\varphi_2]} \text{ (Framing)}$$

Fixpoint Reasoning

$$\frac{\varphi}{\varphi[\psi/X]} \text{ (Set Variable Substitution)}$$

$$\frac{}{\varphi[(\mu X.\varphi/X)] \rightarrow \mu X.\varphi} \text{ (PreFixpoint)}$$

$$\frac{\varphi[\psi/X] \rightarrow \psi}{\mu X.\varphi \rightarrow \psi} \text{ (Knaster-Tarski)}$$

Propositional fragment

$$\frac{}{\varphi \rightarrow (\psi \rightarrow \varphi)} \text{ (Prop 1)}$$

$$\frac{}{(\varphi \rightarrow (\psi \rightarrow \theta)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \theta))} \text{ (Prop 2)}$$

$$\frac{}{((\varphi \rightarrow \perp) \rightarrow \perp) \rightarrow \varphi} \text{ (Prop 3)}$$

$$\frac{\varphi_1 \quad \varphi_1 \rightarrow \varphi_2}{\varphi_2} \text{ (Modus Ponens)}$$

→ No difficulty!

As usual:

```
injective symbol Prf : #Pattern → TYPE;
```

Propositional fragment

- $\frac{}{\varphi \rightarrow (\psi \rightarrow \varphi)}$ (Prop 1)

```
symbol prop-1 :  $\Pi$  ( $\varphi \ \psi$  : #Pattern),  
  Prf ( $\varphi \Rightarrow_{ML} (\psi \Rightarrow_{ML} \varphi)$ );
```

Propositional fragment

- $\frac{}{\varphi \rightarrow (\psi \rightarrow \varphi)}$ (Prop 1)

```
symbol prop-1 :  $\Pi$  ( $\varphi \ \psi$  : #Pattern),  
  Prf ( $\varphi \Rightarrow_{ML} (\psi \Rightarrow_{ML} \varphi)$ );
```

- $\frac{}{(\varphi \rightarrow (\psi \rightarrow \theta)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \theta))}$ (Prop 2)
→ **To be done as an exercise!**

- $\frac{}{((\varphi \rightarrow \perp) \rightarrow \perp) \rightarrow \varphi}$ (Prop 3)
→ **To be done as an exercise!**

- $\frac{\varphi_1 \quad \varphi_1 \rightarrow \varphi_2}{\varphi_2}$ (Modus Ponens)

```
symbol mp :  $\Pi$  ( $\varphi_1 \ \varphi_2$  : #Pattern),  
  Prf  $\varphi_1 \rightarrow$  Prf ( $\varphi_1 \Rightarrow_{ML} \varphi_2$ )  $\rightarrow$  Prf  $\varphi_2$ ;
```

A MATCHING LOGIC proof encoded into DEDUKTI

$$\frac{\frac{\Gamma \vdash \varphi \rightarrow \alpha \quad (P1)}{\Gamma \vdash \varphi \rightarrow (\alpha \rightarrow \varphi)} \quad (P1) \quad \frac{\frac{\Gamma \vdash (\varphi \rightarrow (\alpha \rightarrow \varphi)) \rightarrow ((\varphi \rightarrow \alpha) \rightarrow \alpha) \quad (P2)}{\Gamma \vdash (\varphi \rightarrow \alpha) \rightarrow \alpha} \quad (MP)}{\Gamma \vdash \alpha} \quad (MP)$$

avec $\alpha \equiv \varphi \rightarrow \varphi$

```
symbol imp-identity :  $\Pi$   $\varphi0$ , Prf ( $\varphi0 \Rightarrow_{ML} \varphi0$ ) :=  
   $\lambda$   $\varphi0$ ,  
    mp ( $\varphi0 \Rightarrow_{ML} (\varphi0 \Rightarrow_{ML} \varphi0)$ )  
      ( $\varphi0 \Rightarrow_{ML} \varphi0$ )  
      (prop-1  $\varphi0$   $\varphi0$ )  
      (mp ( $\varphi0 \Rightarrow_{ML} ((\varphi0 \Rightarrow_{ML} \varphi0) \Rightarrow_{ML} \varphi0)$ )  
        (( $\varphi0 \Rightarrow_{ML} (\varphi0 \Rightarrow_{ML} \varphi0)$ )  $\Rightarrow_{ML} (\varphi0 \Rightarrow_{ML} \varphi0)$ )  
        (prop-1  $\varphi0$  ( $\varphi0 \Rightarrow_{ML} \varphi0$ ))  
        (prop-2  $\varphi0$  ( $\varphi0 \Rightarrow_{ML} \varphi0$ )  $\varphi0$ ));
```

$$\frac{}{\varphi[y/x] \rightarrow \exists x.\varphi} \quad (\exists\text{-Quantifier})$$

$$\frac{\varphi_1 \rightarrow \varphi_2 \quad (\text{when } x \notin FV(\varphi_2))}{(\exists x.\varphi_1) \rightarrow \varphi_2} \quad (\exists\text{-Generalization})$$

$$\frac{}{\varphi[y/x] \rightarrow \exists x.\varphi} \quad (\exists\text{-Quantifier})$$

$$\frac{\varphi_1 \rightarrow \varphi_2 \quad (\text{when } x \notin FV(\varphi_2))}{(\exists x.\varphi_1) \rightarrow \varphi_2} \quad (\exists\text{-Generalization})$$

Problems:

- Substitution
- Checking of free variable

$$\frac{}{\varphi[y/x] \rightarrow \exists x.\varphi} \quad (\exists\text{-Quantifier})$$

$$\frac{\varphi_1 \rightarrow \varphi_2 \quad (\text{when } x \notin FV(\varphi_2))}{(\exists x.\varphi_1) \rightarrow \varphi_2} \quad (\exists\text{-Generalization})$$

Problems:

- Substitution
- Checking of free variable

→ Solution: HOAS

FOL reasoning

- $\frac{}{\varphi[y/x] \rightarrow \exists x.\varphi}$ (\exists -Quantifier)

```
symbol ex-quantifier :  
   $\Pi(\varphi : \#Element \rightarrow \#Pattern)$   
   $(y : \#Element),$   
  Prf  $(\varphi y \Rightarrow_{ML} (\exists_{ML} \varphi)) ;$ 
```

Very close to $\frac{}{\varphi \rightarrow \exists x.\varphi}$ (\exists -Quantifier)
because α -renaming is done by the DEDUKTI binder.

FOL reasoning

- $$\frac{}{\varphi[y/x] \rightarrow \exists x.\varphi} \text{ (\exists-Quantifier)}$$

```
symbol ex-quantifier :  
   $\Pi$  ( $\varphi$  : #Element  $\rightarrow$  #Pattern)  
    (y : #Element),  
    Prf ( $\varphi$  y  $\Rightarrow_{ML}$  ( $\exists_{ML}$   $\varphi$ )) ;
```

- $$\frac{\varphi_1 \rightarrow \varphi_2 \quad (\text{when } x \notin FV(\varphi_2))}{(\exists x.\varphi_1) \rightarrow \varphi_2} \text{ (\exists-Generalization)}$$

```
symbol ex-generalization :  
   $\Pi$  ( $\varphi_1$  : #Element  $\rightarrow$  #Pattern)  
    ( $\varphi_2$  : #Pattern),  
    ( $\Pi$  (x : #Element), Prf ( $\varphi_1$  x  $\Rightarrow_{ML}$   $\varphi_2$ ))  
     $\rightarrow$  Prf ( ( $\exists_{ML}$   $\varphi_1$ )  $\Rightarrow_{ML}$   $\varphi_2$  ) ;
```

$$\frac{}{C[\perp] \rightarrow \perp} \text{ (Propagation}_{\perp}\text{)}$$

$$\frac{}{C[\varphi_1 \vee \varphi_2] \rightarrow C[\varphi_1] \vee C[\varphi_2]} \text{ (Propagation}_{\vee}\text{)}$$

$$\frac{\varphi_1 \rightarrow \varphi_2}{C[\varphi_1] \rightarrow C[\varphi_2]} \text{ (Framing)}$$

$$\frac{\text{(when } x \notin FV(C)\text{)}}{C[\exists x.\varphi] \rightarrow \exists x.C[\varphi]} \text{ (Propagation}_{\exists}\text{)}$$

$$\frac{}{C[\perp] \rightarrow \perp} \text{ (Propagation}_{\perp}\text{)}$$

$$\frac{}{C[\varphi_1 \vee \varphi_2] \rightarrow C[\varphi_1] \vee C[\varphi_2]} \text{ (Propagation}_{\vee}\text{)}$$

$$\frac{\varphi_1 \rightarrow \varphi_2}{C[\varphi_1] \rightarrow C[\varphi_2]} \text{ (Framing)}$$

$$\frac{\text{(when } x \notin FV(C)\text{)}}{C[\exists x.\varphi] \rightarrow \exists x.C[\varphi]} \text{ (Propagation}_{\exists}\text{)}$$

Problem:

- Application context $C ::= \square \mid C @ \varphi \mid \varphi @ C$

Frame reasoning - Application context

Model the BNF grammar $C ::= \square \mid C @ \varphi \mid \varphi @ C$:

```
symbol #AC : TYPE ;  
symbol HOLE : #AC ;  
symbol ACleft : #AC → #Pattern → #AC ;  
symbol ACright : #Pattern → #AC → #AC ;
```

Frame reasoning - Application context

Model the BNF grammar $C ::= \square \mid C @ \varphi \mid \varphi @ C$:

```
symbol #AC : TYPE ;
symbol HOLE : #AC ;
symbol ACleft : #AC → #Pattern → #AC ;
symbol ACright : #Pattern → #AC → #AC ;
```

Translate an application context into a pattern:

```
symbol AC2P : #AC → #Pattern → #Pattern ;
rule AC2P HOLE $x ↦ $x ;
rule AC2P (ACleft $C $P) $x ↦ (AC2P $C $x) @ML $P ;
rule AC2P (ACright $P $C) $x ↦ $P @ML (AC2P $C $x) ;
```


Frame reasoning - Application context

Model the BNF grammar $C ::= \square \mid C @ \varphi \mid \varphi @ C$:

```
symbol #AC : TYPE ;
symbol HOLE : #AC ;
symbol ACleft : #AC → #Pattern → #AC ;
symbol ACright : #Pattern → #AC → #AC ;
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Translate an application context into a pattern:

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symbol AC2P : #AC → #Pattern → #Pattern ;
rule AC2P HOLE $x ↦ $x ;
rule AC2P (ACleft $C $P) $x ↦ (AC2P $C $x) @ML $P ;
rule AC2P (ACright $P $C) $x ↦ $P @ML (AC2P $C $x) ;
```

Translate the rule $\frac{}{C[\perp] \rightarrow \perp}$ (*Propagation_⊥*):

```
symbol propag-bot :
  Π(C : #AC), Prf (AC2P C ⊥ML ⇒ML ⊥ML) ;

type propag-bot (ACright (injSym •) HOLE) ;
  // Prf ((injSym •) @ML ⊥ML ⇒ML ⊥ML)
```

- $\frac{}{C[\perp] \rightarrow \perp}$ (*Propagation $_{\perp}$*) → **Already done!**
- $\frac{}{C[\varphi_1 \vee \varphi_2] \rightarrow C[\varphi_1] \vee C[\varphi_2]}$ (*Propagation $_{\vee}$*)
→ **To be done as an exercise!**
- $\frac{\varphi_1 \rightarrow \varphi_2}{C[\varphi_1] \rightarrow C[\varphi_2]}$ (*Framing*) → **To be done as an exercise!**
- $\frac{(when\ x \notin FV(C))}{C[\exists x.\varphi] \rightarrow \exists x.C[\varphi]}$ (*Propagation $_{\exists}$*)
→ **Combine HOAS + Application context**

$$\frac{\varphi}{\varphi[\psi/X]} \text{ (Set Variable Substitution)}$$

$$\frac{}{\varphi[(\mu X.\varphi)/X] \rightarrow \mu X.\varphi} \text{ (PreFixpoint)}$$

$$\frac{\varphi[\psi/X] \rightarrow \psi}{(\mu X.\varphi) \rightarrow \psi} \text{ (Knaster-Tarski)}$$

Problem:

- Is there a problem?

Fixpoint reasoning

- $\frac{\varphi}{\varphi[\psi/X]}$ (Set Variable Substitution)
- $\frac{}{\varphi[(\mu X.\varphi)/X] \rightarrow \mu X.\varphi}$ (PreFixpoint)

```
symbol Pre-fixpoint :  
   $\Pi (\varphi : \#Pattern \rightarrow \#Pattern),$   
  Prf (  $\varphi (\mu_{ML} \varphi) \Rightarrow_{ML} (\mu_{ML} \varphi)$  ) ;
```

- $\frac{\varphi[\psi/X] \rightarrow \psi}{(\mu X.\varphi) \rightarrow \psi}$ (Knaster-Tarski)

```
symbol Knaster-Tarski :  
   $\Pi (\varphi : \#Pattern \rightarrow \#Pattern)$   
  (  $\psi : \#Pattern$  ),  
  Prf (  $\varphi \psi \Rightarrow_{ML} \psi$  )  $\rightarrow$   
  Prf (  $(\mu_{ML} \varphi) \Rightarrow_{ML} \psi$  ) ;
```

```
symbol  $\mu_{ML}$  : (  $\#Pattern \rightarrow \#Pattern$  )  $\rightarrow \#Pattern$  ;
```

Fixpoint reasoning

- $\frac{\varphi}{\varphi[\psi/X]}$ (Set Variable Substitution) \rightarrow **X is a free variable!**
- $\frac{}{\varphi[(\mu X.\varphi)/X] \rightarrow \mu X.\varphi}$ (PreFixpoint)

```
symbol Pre-fixpoint :  
   $\Pi (\varphi : \#Pattern \rightarrow \#Pattern),$   
  Prf (  $\varphi (\mu_{ML} \varphi) \Rightarrow_{ML} (\mu_{ML} \varphi)$  ) ;
```

- $\frac{\varphi[\psi/X] \rightarrow \psi}{(\mu X.\varphi) \rightarrow \psi}$ (Knaster-Tarski)

```
symbol Knaster-Tarski :  
   $\Pi (\varphi : \#Pattern \rightarrow \#Pattern)$   
   $(\psi : \#Pattern),$   
  Prf (  $\varphi \psi \Rightarrow_{ML} \psi$  )  $\rightarrow$   
  Prf (  $(\mu_{ML} \varphi) \Rightarrow_{ML} \psi$  ) ;
```

```
symbol  $\mu_{ML}$  : ( $\#Pattern \rightarrow \#Pattern$ )  $\rightarrow$   $\#Pattern$  ;
```

The last problem

- $\frac{\varphi}{\varphi[\psi/X]}$ (Set Variable Substitution)

```
symbol Set-var-subst :  
  Π (φ ψ : #Pattern) (n : nat),  
    Prf φ → Prf (subst φ ψ n) ;
```

where the free variable is modelled by:

```
symbol Free : nat → #Set ;
```

and the substitution is modelled by:

```
symbol subst :  
  #Pattern → #Pattern → nat → #Pattern; // φ[ψ/X]  
  
rule subst (injEl $x) _ _ ↦ injEl $x;  
  
rule subst (injSet (Free $m)) $ψ $n ↦  
  ite (eq $m $n) $ψ (injSet (Free $m));
```

$$\frac{}{\exists x.x} \text{ (Existence)}$$

$$\frac{}{\neg(C_1[x \wedge \varphi] \wedge C_2[x \wedge \neg\varphi])} \text{ (Singleton)}$$

To be done as an exercise!

① A shallow encoding to execute a program in DEDUKTI

② A deep encoding to check proofs in DEDUKTI

Translate MATCHING LOGIC constructors, notations and symbols

Translate MATCHING LOGIC proof system

③ Conclusion

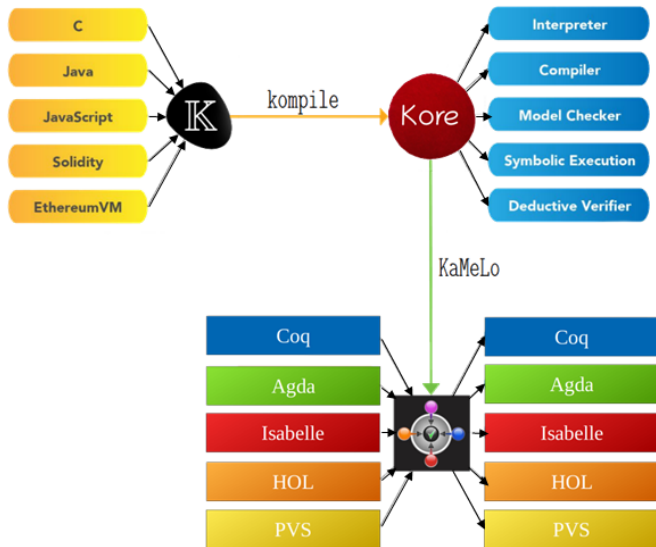
Computational part of an embedding

- Use rewriting rules!
 - Be careful about the expressivity of rewriting system!
 - Be careful to keep the confluence!
 - Be careful to keep the terminaison!

Axiomatic part of an embedding

- Model the provability relation: `symbol` $\text{Prf} : \# \text{Pattern} \rightarrow \text{TYPE}$
- Model variables and binders: HOAS vs De Bruijn indices
- Model grammar:
 - type as set
 - symbol as constructor

KAMELO in action



Typing

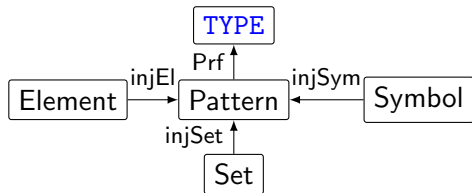
MATCHING LOGIC defines patterns φ .

- $\varphi ::= x \mid X \mid \sigma \mid \varphi \varphi \mid \perp \mid \varphi \rightarrow \varphi \mid \exists x.\varphi \mid \mu X.\varphi$

```
symbol @ML : #Pattern → #Pattern → #Pattern;  
notation @ML infix left 5;  
symbol ⇒ML : #Pattern → #Pattern → #Pattern;  
symbol ∃ML : (#Element → #Pattern) → #Pattern;  
symbol μML : (#Pattern → #Pattern) → #Pattern;
```

```
symbol #Pattern : TYPE;  
symbol #Element : TYPE;  
symbol #Set : TYPE;  
symbol #Symbol : TYPE;
```

```
symbol injEl : #Element → #Pattern;  
symbol injSet : #Set → #Pattern;  
symbol injSym : #Symbol → #Pattern;
```



Extension of $\mathcal{L}_{MiniExp}$: \mathcal{L}_{IMP}

<pre> syntax AExp ::= Int Id AExp "/" AExp > AExp "+" AExp "(" AExp ")" </pre>	<pre> [left, strict] [left, strict] [bracket] </pre>
<pre> syntax BExp ::= Bool AExp "<" AExp "not" BExp > BExp "and" BExp "(" BExp ")" </pre>	<pre> [seqstrict] [strict] [left, strict(1)] [bracket] </pre>

Extension of $\mathcal{L}_{MiniExp}$: \mathcal{L}_{IMP}

<code>syntax AExp ::= Int Id</code> <code>AExp "/" AExp</code> > <code>AExp "+" AExp</code> <code>"(" AExp ")"</code>	<code>[left, strict]</code> <code>[left, strict]</code> <code>[bracket]</code>
<code>syntax BExp ::= Bool</code> <code>AExp "<" AExp</code> <code>"not" BExp</code> > <code>BExp "and" BExp</code> <code>"(" BExp ")"</code>	<code>[seqstrict]</code> <code>[strict]</code> <code>[left, strict(1)]</code> <code>[bracket]</code>

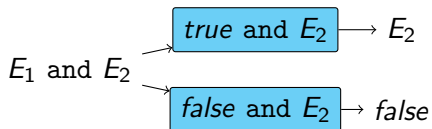
- There are 2 ways to define an evaluation strategy:
 - Context
 - Attributes `strict` and `seqstrict`
- Everything is compiled in the same mechanism based on rewriting rules.

Evaluation strategy

- Example: BExp "and" BExp [left, strict(1)]

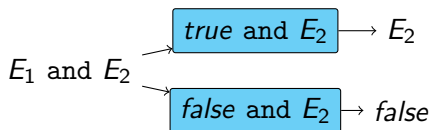
Evaluation strategy

- Example: BExp "and" BExp [left, strict(1)]



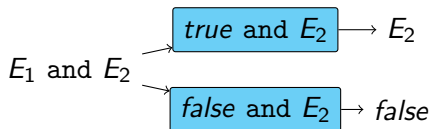
Evaluation strategy

- **Example: BExp "and" BExp [left, strict(1)]**
 1. rule $E_1 \text{ and } E_2 \Rightarrow E_1 \curvearrowright (*_{\text{and}}^1 E_2)$ requires $E_1 \notin \text{KResult}$
 2. rule $E_1 \curvearrowright (*_{\text{and}}^1 E_2) \Rightarrow E_1 \text{ and } E_2$ requires $E_1 \in \text{KResult}$



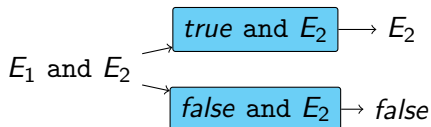
Evaluation strategy

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 3. rule $\text{true and } b \Rightarrow b$
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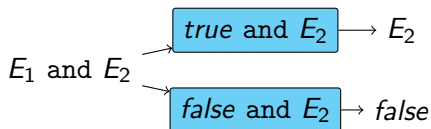
Evaluation strategy

- **Example: BExp "and" BExp [left, strict(1)]**
 1. rule $\langle E_1 \text{ and } E_2 \rightsquigarrow s \rangle_{km}$
 $\Rightarrow \langle E_1 \rightsquigarrow (*_{\text{and}}^1 E_2) \rightsquigarrow s \rangle_{km}$ requires $E_1 \notin \text{KResult}$
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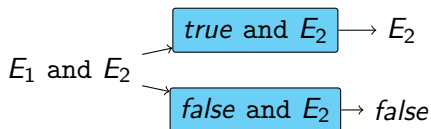
Evaluation strategy

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Evaluation strategy

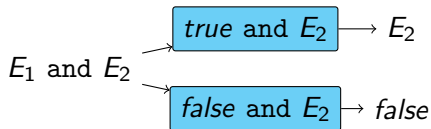
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Evaluation strategy

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Evaluation strategy

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 4. rule $\langle \text{false and } _ \rightsquigarrow s \rangle_{km} \Rightarrow \langle \text{false} \rightsquigarrow s \rangle_{km}$
- $\langle (\text{true and false}) \text{ and } (\text{true and true}) \rightsquigarrow _ \rangle_k \cdot \text{Map}$

Evaluation strategy

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$\langle (\text{true and false}) \text{ and } (\text{true and true}) \curvearrowright . \rangle_k . \text{Map}$

$\hookrightarrow_1 \langle (\text{true and false}) \curvearrowright (*_{\text{and}}^1 (\text{true and true})) \curvearrowright . \rangle_k . \text{Map}$

Evaluation strategy

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$\hookrightarrow_3 \langle \text{false} \curvearrowright (*_{\text{and}}^1 (\text{true and true})) \curvearrowright . \rangle_k . \text{Map}$

Evaluation strategy

- **Example: BExp "and" BExp** [left, strict(1)]

1. rule $\langle E_1 \text{ and } E_2 \rightsquigarrow s \rangle_{km}$
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$\langle (\text{true and false}) \text{ and } (\text{true and true}) \rightsquigarrow . \rangle_k . \text{Map}$

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Evaluation strategy

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$\langle (\text{true and false}) \text{ and } (\text{true and true}) \curvearrowright . \rangle_k . \text{Map}$
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 $\hookrightarrow_2 \langle \text{false and } (\text{true and true}) \curvearrowright . \rangle_k . \text{Map}$
 $\hookrightarrow_4 \langle \text{false} \curvearrowright . \rangle_k . \text{Map}$

Evaluation strategy

- **Example: BExp "and" BExp** [left, strict(1)]

1. rule $\langle E_1 \text{ and } E_2 \rightsquigarrow s \rangle_{km}$
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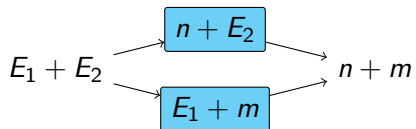
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$\langle (\text{true and false}) \text{ and } (\text{true and true}) \rightsquigarrow . \rangle_k . \text{Map}$
 $\hookrightarrow_1 \langle (\text{true and false}) \rightsquigarrow (*_{\text{and}}^1 (\text{true and true})) \rightsquigarrow . \rangle_k . \text{Map}$
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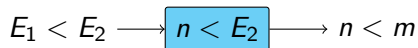
- **K computation:** a list ($\text{List}\{K, \rightsquigarrow\}$), potentially nested, of computations to be performed sequentially.
- **The sort KResult,** to distinguish the (final) values of expressions (Here, $\text{syntax KResult} ::= \text{Int} \mid \text{Bool}$).

Evaluation strategy

Attribute **strict**

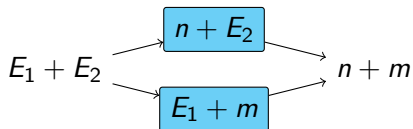


Attribute **seqstrict**



Evaluation strategy

Attribute **strict**



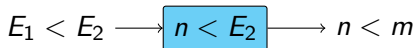
rule $E_1 + E_2 \Rightarrow E_1 \curvearrowright (*_+^1 E_2)$
requires $E_1 \notin \text{KResult}$

rule $E_1 \curvearrowright (*_+^1 E_2) \Rightarrow E_1 + E_2$
requires $E_1 \in \text{KResult}$

rule $E_1 + E_2 \Rightarrow E_2 \curvearrowright (*_+^2 E_1)$
requires $E_2 \notin \text{KResult}$

rule $E_2 \curvearrowright (*_+^2 E_1) \Rightarrow E_1 + E_2$
requires $E_2 \in \text{KResult}$

Attribute **seqstrict**



rule $E_1 < E_2 \Rightarrow E_1 \curvearrowright (*_<^1 E_2)$
requires $E_1 \notin \text{KResult}$

rule $E_1 \curvearrowright (*_<^1 E_2) \Rightarrow E_1 < E_2$
requires $E_1 \in \text{KResult}$

rule $E_1 < E_2 \Rightarrow E_2 \curvearrowright (*_<^2 E_1)$
requires $E_2 \notin \text{KResult}$

$\wedge E_1 \in \text{KResult}$
rule $E_2 \curvearrowright (*_<^2 E_1) \Rightarrow E_1 < E_2$
requires $E_2 \in \text{KResult}$

Overview of MATCHING LOGIC and KORE

- MATCHING LOGIC defines patterns φ .
 - $\varphi ::= x \mid X \mid \sigma \mid \varphi \varphi \mid \perp \mid \varphi \rightarrow \varphi \mid \exists x.\varphi \mid \mu X.\varphi$
 - A pattern is interpreted as the set of elements that it matches.
- KORE is a theory of MATCHING LOGIC
 - = Theory of sorts
 - + Theory of rewriting
 - + Theory of equality

Overview of KPROVER

- Parametrized by a \mathbb{K} semantic
- Based on the REACHABILITY LOGIC
 - an extension of Hoare logic and separation logic
- Reachability property $\varphi \rightsquigarrow \varphi'$:
 - During the execution of a program,
if φ is **matched**, then φ' will be **matched** later on in a finite number of steps, or there is divergence.
- Example:
 $(N \geq 0) \wedge (S \geq 0) \wedge$
 $\langle\langle \text{while } 0 < n \text{ do } \{ s = s + n ; n = n - 1 ; \} \rangle_k \langle n \mapsto N, s \mapsto S \rangle_{env} \rangle$
 $\rightsquigarrow \langle\langle \cdot \rangle_k \langle n \mapsto 0, s \mapsto S + \frac{N*(N+1)}{2} \rangle_{env} \rangle$

Different rules for the same semantics

A. **rule** $\langle\langle x = i; \curvearrowright s \rangle_k \langle m (x \mapsto -) \rangle_{env}\rangle$
 $\Rightarrow \langle\langle s \rangle_k \langle m (x \mapsto i) \rangle_{env}\rangle$

+ Implementation of Map in DEDUKTI (so without ACUI)

B. **rule** $\langle\langle x = i; \curvearrowright s \rangle_k \langle m \rangle_{env}\rangle$
 $\Rightarrow \langle\langle s \rangle_k \langle \text{update } m \ x \ i \rangle_{env}\rangle$

+ Implementation of Map in DEDUKTI (so without ACUI)

C. **rule** $\langle\langle x = i; \Rightarrow s \dots \rangle_k \langle \dots (x \mapsto (- \Rightarrow i)) \rangle_{env}\rangle$

+ Need to translate towards the rule B.³

³Work in progress of Everett Hildenbrandt.

Understand the problem

Is the rule:

$$\begin{aligned} \text{rule } & \langle \langle x = i; \curvearrowright s \rangle_k \langle m (x \mapsto -) \rangle_{env} \rangle \\ & \Rightarrow \langle \langle s \rangle_k \langle m (x \mapsto i) \rangle_{env} \rangle \end{aligned}$$

applies in the following cases?

Understand the problem

Is the rule:

$$\begin{aligned} \text{rule } & \langle \langle x = i; \curvearrowright s \rangle_k \langle m (x \mapsto -) \rangle_{env} \rangle \\ & \Rightarrow \langle \langle s \rangle_k \langle m (x \mapsto i) \rangle_{env} \rangle \end{aligned}$$

applies in the following cases?

$$\textcircled{1} \langle \langle x = 10; \curvearrowright . \rangle_k \langle (y \mapsto 52) (x \mapsto 42) \rangle_{env} \rangle$$

Understand the problem

Is the rule:

$$\begin{aligned} \text{rule } & \langle \langle x = i; \curvearrowright s \rangle_k \langle m (x \mapsto -) \rangle_{env} \rangle \\ & \Rightarrow \langle \langle s \rangle_k \langle m (x \mapsto i) \rangle_{env} \rangle \end{aligned}$$

applies in the following cases?

- 1 $\langle \langle x = 10; \curvearrowright . \rangle_k \langle (y \mapsto 52) (x \mapsto 42) \rangle_{env} \rangle$
- 2 $\langle \langle x = 10; \curvearrowright . \rangle_k \langle (x \mapsto 42) (y \mapsto 52) \rangle_{env} \rangle$

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- 3 $\langle\langle x = 10; \curvearrowright . \rangle_k \langle (x \mapsto 42) \rangle_{env} \rangle$
- 4 $\langle\langle x = 10; \curvearrowright . \rangle_k \langle .\text{Map} (x \mapsto 42) \rangle_{env} \rangle$

Reminder: `.Map` = Empty environment

Understand the problem

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- 3 $\langle\langle x = 10; \curvearrowright . \rangle_k \langle (x \mapsto 42) \rangle_{env} \rangle$
- 4 $\langle\langle x = 10; \curvearrowright . \rangle_k \langle \text{.Map } (x \mapsto 42) \rangle_{env} \rangle$
- 5 $\langle\langle x = 10; \curvearrowright . \rangle_k \langle (x \mapsto 42) \text{.Map} \rangle_{env} \rangle$

Reminder: **.Map** = Empty environment

Understand the problem

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applies in the following cases?

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- 6 $\langle\langle x = 10; \curvearrowright . \rangle_k \langle \text{.Map } (x \mapsto 42) \text{.Map } \text{.Map} \text{.Map} \rangle_{env} \rangle$

Reminder: **.Map** = Empty environment

Assessment & scaling up

- ✓ Syntax
- ✓ Configuration
- ✓ Evaluation strategy (context, attributes `strict` and `seqstrict`)
- ✓ Rewriting rule (conditional or not, attribute `owise`)

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- ✗ Rewriting modulo ACUI
- ✗ \mathbb{K} standard library

Assessment & scaling up

Semantics	Last update	KORE file	DEDUKTI file	DEDUKTI check	Execution
Ethereum Virtual Machine (EVM)	2022	✓ (60k)	✓	✗	-
Michelson	2022	✓ (36k)	✓	✗	-
C	2022	✗	-	-	-
IELE	Dec 2021	-	-	-	-
WebAssembly	2022	✓ (30k)	✓	✗	-
Elrond	April 2021	✗	-	-	-
P4	May 2021	✓ (86k)	✓	✗	-
Plutus core	2022	✓ (40k)	✓	-	-
Java	Sept 2021	✗	-	-	-
Boogie	Sept 2021	✗	-	-	-
x86-64	2020	✗	-	-	-
Ewasm	2020	✗	-	-	-
Hybrid programs	2020	✗	-	-	-
K	2020	-	-	-	-
Yul	2019	✗	-	-	-
Ethereum Environment Interface (EEI)	2019	-	-	-	-
LLVM	2018	✗	-	-	-
Vyper	2018	-	-	-	-
ERC20	2018	-	-	-	-
ERC777	2018	-	-	-	-
Solidity	2017	-	-	-	-
Orc	2017	✗	-	-	-
Haskell core	2017	✗	-	-	-
Cink	2015	✗	-	-	-
JavaScript	2015	✗	-	-	-
JVM	2014	✗	-	-	-
JavaCard	2014	✗	-	-	-
Alk	2014	✗	-	-	-
AADL	2013	✗	-	-	-
Modelink	2013	✗	-	-	-
Python	2013	✗	-	-	-
OCaml	2013	✗	-	-	-