Langages Formels – TD 4

Guillaume Scerri
guillaume.scerri@lmf.cnrs.fr

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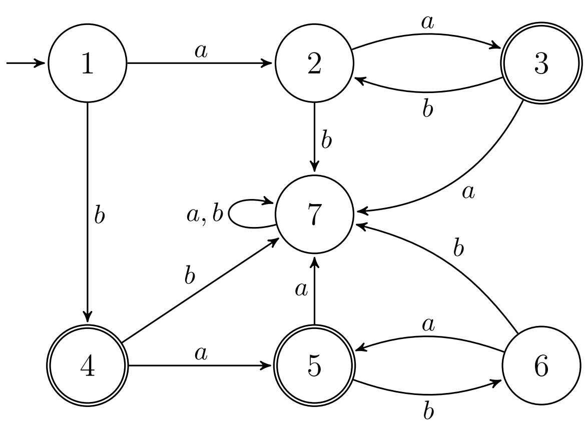
Exercise 1 : Regular identities

We study identities on regular expressions r, s, t. Here, $r = s$ means $\mathcal{L}(r) = \mathcal{L}(s)$.

1. Prove the following identities :
   (a) $(r + s)t = rt + st$
   (b) $(r^*)^* = r^*$
   (c) $(rs + r)^*r = r(sr + r)^*$
2. Prove or disprove the following identities :
   (a) $(r + s)^* = r^* + s^*$
   (b) $(r^*s^*)^* = (r + s)^*$
   (c) $s(rs + s)^*r = rr^*s(rr^*s)^*$

Exercise 2 : Minimization by Moore's algorithm

1. Minimize the automata $\mathcal{A}_1$ and $\mathcal{A}_2$, using Moore's algorithm :



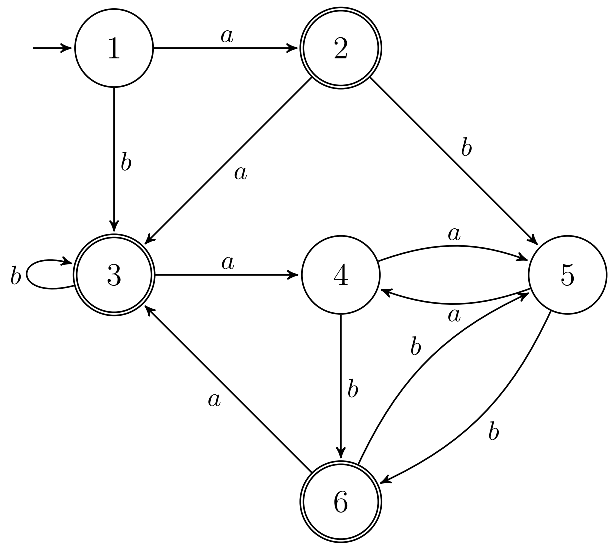
(a) Automaton $\mathcal{A}_1$

2. Give a minimal automaton for $\mathcal{L} = ((a(a + b)^2 + b)^*a(a + b))^*$.

Exercise 3 :

We define by $|e|$ the size of a regular expression e, i.e. the number of symbols appearing in e.

1. Is there a unique minimal regular expression e such that $\mathcal{L}(e) = L$ for a given regular language L ?
2. Give a procedure which, given a finite automaton $\mathcal{A}$, returns a minimal regular expression e such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(e)$.



(a) Automaton $\mathcal{A}_2$

Exercise 4 : Flashback

We proved that the language of palindromes over alphabet $\Sigma = \{a, b\}$ is not recognizable. We say a palindrome is *non trivial* if its length is greater than or equal to 2. Determine which of the following languages are recognizable (and prove it) :

1. The language L_1 of words in Σ^* containing a non trivial palindrome as a prefix.
2. The language L_2 of words in Σ^* containing a non trivial palindrome of even length as a prefix.

Exercise 5 : Résiduels

Calculer les résiduels de $\mathcal{L} = a^*(aa + b) + b(a + ba)^*$ et construire son automate minimal.

Exercise 6 : Characterizing recognizability

We want to show a converse to the pumping lemma. We say that a language L satisfies P_h if for all $uv_1 \dots v_h w$ avec $|v_i| \geq 1$, there exists $0 \leq j < k \leq h$ such that

$$uv_1 \dots v_h w \in L \Leftrightarrow uv_1 \dots v_j v_{k+1} \dots v_h w \in L.$$

The theorem of Ehrenfeucht, Parikh & Rozenberg states that L is rational iff there exists h such that L satisfies P_h.

1. Show that if L satisfies P_h, then $w^{-1}L$ also does for every word $w \in \Sigma^*$.
2. Let $h \in \mathbb{N}$. We want to show that the number of languages satisfying P_h is finite. We use the following statement of Ramsey's theorem :

   > For every k there is N such that, for every set E of cardinal greater than N and every bipartition $\mathcal{P}$ of $\mathfrak{P}_2(E) = \{ \{e, e'\} \ : \ e, e' \in E, \ e \neq e' \}$, there exists a subset $F \subseteq E$ of cardinal k such that $\mathfrak{P}_2(F)$ is contained in one of the classes of $\mathcal{P}$.

   Let N be the natural number given by Ramsey's theorem for $k = h+1$. Let L and L' be two languages satisfying P_h and coinciding on words of size smaller than N. Prove that they coincide on words or size $M \geq N$, by induction on M. You may consider, for a word $f = a_1 \dots a_N t$ of size M (with $a_i \in \Sigma$), the following partition of $\mathfrak{P}_2([0; N])$:

   $$X_f = \{ \ (j, k) \ : \ 0 \leq j < k \leq N, \ a_1 \dots a_j a_{k+1} \dots a_N t \in L \ \}$$
   $$Y_f = \mathfrak{P}_2([0; N]) \setminus X_f$$

   Conclude.
3. Conclude that if a language L satisfies P_h for some h, then L is regular.