Langages Formels
TD 5

Guillaume Scerri
guillaume.scerri@lmf.cnrs.fr

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Exercise 1 : State complexity of a language

Given a recognizable language L, we define its *state complexity* $\mathsf{Sc}(L)$ by the number of states of its minimal automaton. Show that the following inequalities hold (L^{t} is the transposed of L, the language of the mirror images of words of L) :

1. $\mathsf{Sc}(L \cap K) \leq \mathsf{Sc}(L)\mathsf{Sc}(K)$;
2. $\mathsf{Sc}(L \cup K) \leq \mathsf{Sc}(L)\mathsf{Sc}(K)$;
3. $\mathsf{Sc}(L^{\mathsf{t}}) \leq 2^{\mathsf{Sc}(L)}$;
4. $\mathsf{Sc}(LK) \leq (2\mathsf{Sc}(L) - 1)2^{\mathsf{Sc}(K)-1}$.

We will now show that some of these bounds have the right order of magnitude. Let $\Sigma = \{ a,\ b \}$.

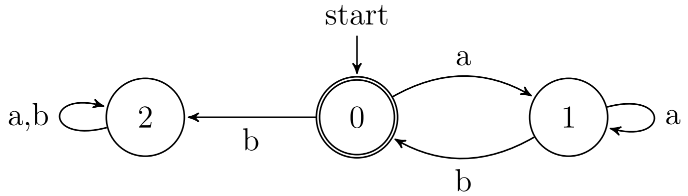
5. Consider $L_n = \{|w|_a + |w|_b = 2n\}$ and $L'_n = \{|w|_a + 2|w|_b = 3n\}$ for the bound for intersection.
6. Consider $L_n = \Sigma^{n-1}a\Sigma^*$ for the bound for transposition.

Exercise 2 : Minimization by Brzozowski inversion

1. Show that the determinized of a co-deterministic co-accessible automaton which recognizes a language L is (isomorphic to) the minimal automaton of L.
2. Using this result, devise a procedure to minimize an automaton. What is the complexity of this method ?

Exercise 3 : Transition monoid

We consider the following finite deterministic complete automaton $\mathcal{A}$ over $\Sigma = \{a, b\}$:



1. Give $\mathcal{L}(\mathcal{A})$.
2. Give M the transition monoid of this automata, a morphism ϕ and $P \subset M$ such that $L = \phi^{-1}(P)$.

Exercise 4 : Monoïde → Automate

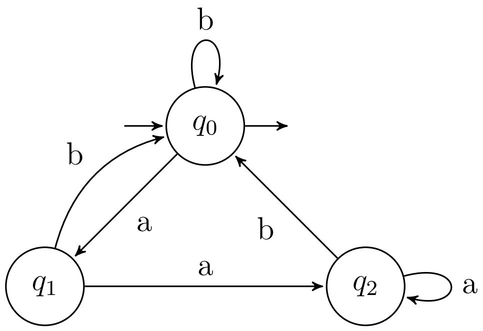
Soit $M = \{ A, B, C, D, E \}$ avec E l'élément neutre et

$u \setminus v$	A	B	C	D	E
A	E	D	C	C	A
B	D	B	C	C	B
C	C	C	B	C	C
D	D	C	C	D	D
E	A	B	C	D	E

avec $\phi(a) = A$, $\phi(b) = B$ et $\phi(c) = C$. Calculer $\phi^{-1}(X)$ pour tout $X \in M$ et tracer l'automate reconnaissant $\phi^{-1}(C)$.

Exercise 5 : Automate → Monoïde

Donnez le monoïde syntaxique M du langage $\mathcal{L}$ reconnu par cet automate, un morphisme ϕ et $P \subset M$ tel que $\phi^{-1}(P) = \mathcal{L}$.



Quelle est la congruence syntaxique de $\mathcal{L}$? Quelles sont ses classes d'équivalence ?