Langages Formels - TD 5

March 4, 2024

Exercise 1 : Flashback

We proved that the language of palindromes over alphabet $\Sigma = \{a, b\}$ is not recognizable. We say a palindrome is *non trivial* if its length is greater than or equal to 2. Determine which of the following languages are recognizable (and prove it):

1. The language L_1 of words in Σ^* containing a non trivial palindrome as a prefix.
2. The language L_2 of words in Σ^* containing a non trivial palindrome of even length as a prefix.

Exercise 2 : Congruences and monoids

An equivalence relation R on Σ^* is a *congruence* if uRv implies $xuyRxvy$ for all x, y. We will call *congruence classes* the equivalence classes of a congruence.

1. Prove that a language is regular iff it is the union of some of the congruence classes of a congruence relation of *finite index*, i.e. with a finite number of congruence classes.

A congruence c_1 is *coarser* (i.e. "grossière") than another congruence c_2 if every congruences classe of c_2 is included in a congruence class of c_1.

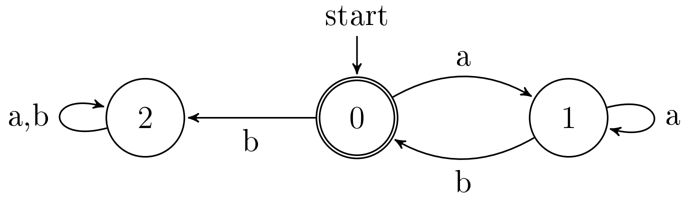
2. Let L be a language. Find a characterization of the coarsest congruence $\equiv_L$ such that L is the union of some of its congruence classes.

This congruence is called the *syntactic* congruence of L.

3. Give a more precise criterion for the recognizability of a language. Apply it to prove that $\{\ a^n b^n\ :\ n \in \mathbb{N}\ \}$ is not recognizable.
4. We know that a language of Σ^* is regular iff there exists a finite monoid $(M, \times)$, a morphism $\mu : (\Sigma^*, \cdot) \to (M, \times)$, and a set $P \subseteq M$ such that $L = \mu^{-1}(P)$. Find a characterization of the smallest such monoid for a regular language L.
5. What is the link between the syntactic congruence, this smallest monoid, and the minimal automaton?

Exercise 3 : Transition monoid

We consider the following finite deterministic complete automaton $\mathcal{A}$ over $\Sigma = \{a, b\}$:



1. Give $\mathcal{L}(\mathcal{A})$.
2. Give M the transition monoid of this automata, a morphism ϕ and $P \subset M$ such that $L = \phi^{-1}(P)$.
3. What is the syntactical congruence of L? What are its equivalence classes?

Exercise 4 : State complexity of a language

Given a recognizable language L, we define its *state complexity* $\mathsf{Sc}(L)$ by the number of states of its minimal automaton. Show that the following inequalities hold (L^{t} is the transposed of L, the language of the mirror images of words of L):

1. $\mathsf{Sc}(L \cap K) \leq \mathsf{Sc}(L)\mathsf{Sc}(K)$;
2. $\mathsf{Sc}(L \cup K) \leq \mathsf{Sc}(L)\mathsf{Sc}(K)$;
3. $\mathsf{Sc}(L^{\mathsf{t}}) \leq 2^{\mathsf{Sc}(L)}$;
4. $\mathsf{Sc}(LK) \leq (2\mathsf{Sc}(L) - 1)2^{\mathsf{Sc}(K)-1}$.

We will now show that some of these bounds have the right order of magnitude. Let $\Sigma = \{ a, b \}$.

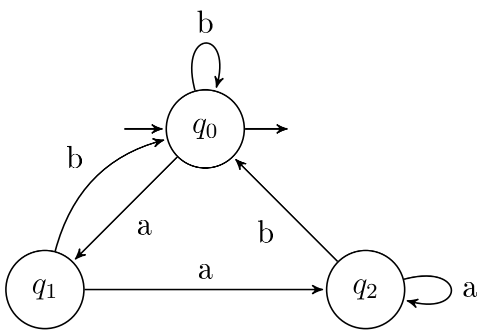
5. Consider $L_n = \{|w|_a + |w|_b = 2n\}$ and $L'_n = \{|w|_a + 2|w|_b = 3n\}$ for the bound for intersection.
6. Consider $L_n = \Sigma^{n-1}a\Sigma^*$ for the bound for transposition.

Contrôle continu 5

À rendre pour le 07/03 à 16h15.

Exercise 5 : Automate → Monoïde

Donnez le monoïde syntaxique M du langage $\mathcal{L}$ reconnu par cet automate, un morphisme ϕ et $P \subset M$ tel que $\phi^{-1}(P) = \mathcal{L}$.



Quelle est la congruence syntaxique de $\mathcal{L}$? Quelles sont ses classes d'équivalence ?

Exercise 6 : Minimization by Brzozowski inversion

1. Show that the determinized of a co-deterministic co-accessible automaton which recognizes a language L is (isomorphic to) the minimal automaton of L.
2. Using this result, devise a procedure to minimize an automaton. What is the complexity of this method ?