Langages Formels

TD 7 - Révisions

Isa Vialard

vialard@lsv.fr

March 14, 2024

Exercise 1 : Intermède: congruences

Let's explore the fabulous world of congruences!

1. Using congruences, prove that $\{ xcx \ : \ x \in \{ a, b \}^* \}$ is not regular.
2. Same question for any infinite subset of $\{ a^n b^n \ : \ n \in \mathbb{N} \}$.
3. Consider the regular language L represented by $a^*b^* + b^*a^*$.
   (a) Draw the minimal automaton for L.
   (b) Give a regular expression describing each of the equivalence classes of the syntactic congruence of L, denoted $\equiv_L$.
4. Let Σ be an alphabet. Let $\equiv$ be a a congruence of finite index over Σ^*. Prove that any equivalence class of $\equiv$ is a regular language of Σ^*.

Exercise 2 : Two way automata (Boustrophédon)

A two way automaton is a finite automaton which, for each transition, can move its reading head one step to the right or one step to the left. Equivalently, it is a Turing machine with one ribbon which cannot write.

1. Build a two way automaton with $O(n)$ states that accept $\Sigma^* a \Sigma^n$.
2. Show that all language accepted by a deterministic two way automaton is regular.
3. Show that from any deterministic two way automaton with n states, we can construct an equivalent deterministic finite automaton with $2^{O(n^2)}$ states.

Exercise 3 : Selection property

A morphism $\mu : A^* \to B^*$ has the *selection property* iff for every regular language L, there exists a regular language $K \subseteq L$ such that μ is injective over K and $\mu(K) = \mu(L)$. The goal of this exercise is to show that every morphism has the selection property.

1. Show that all injective morphisms have the selection property.
2. Show that if morphisms μ and ν have the selection property, then the morphism $\mu \circ \nu$ also has it.

We call *projection* a morphism $\pi : A^* \to B^*$ such that for every letter $a \in A$, $\pi(a) = a$ or $\pi(a) = \varepsilon$.

3. Show that for every morphism $\mu : A^* \to B^*$, there exists an alphabet C, an injective morphism $\iota : A^* \to C^*$ and a projection $\pi : C^* \to B^*$ such that $\mu = \pi \circ \iota$.

We call *elementary projection* a projection $\pi : A^* \to B^*$ such that there exists a unique letter $a \in A$ such that $\pi(a) = \varepsilon$.

4. Show that every projection is the composition of elementary projections.
5. Show that all elementary projection has the selection property. *(Cette question est plus dure qu'il n'y parait.)*
6. Conclude.