On the Probabilistic and Statistical Verification of Infinite Markov Chains

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CNRS, ENS Paris-Saclay
France

Joint work with Benoît Barbot (LACL) and Serge Haddad (LMF)

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General purpose

Design algorithms to estimate probabilities in some **infinite-state** Markov chains, **with guarantees**

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Our contributions

- Review two existing approaches (approximation algorithm and estimation algorithm) and specify the required hypothesis for correctness
- Propose an approach based on **importance sampling** and **abstraction** to partly relax the hypothesis
- Analyze empirically the approaches

Discrete-time Markov chains

Discrete-time Markov chain (DTMC)

$\mathscr{C} = (S, s_0, \delta)$ with S at most denumerable, $s_0 \in S$ and $\delta : S \to \mathrm{Dist}(S)$

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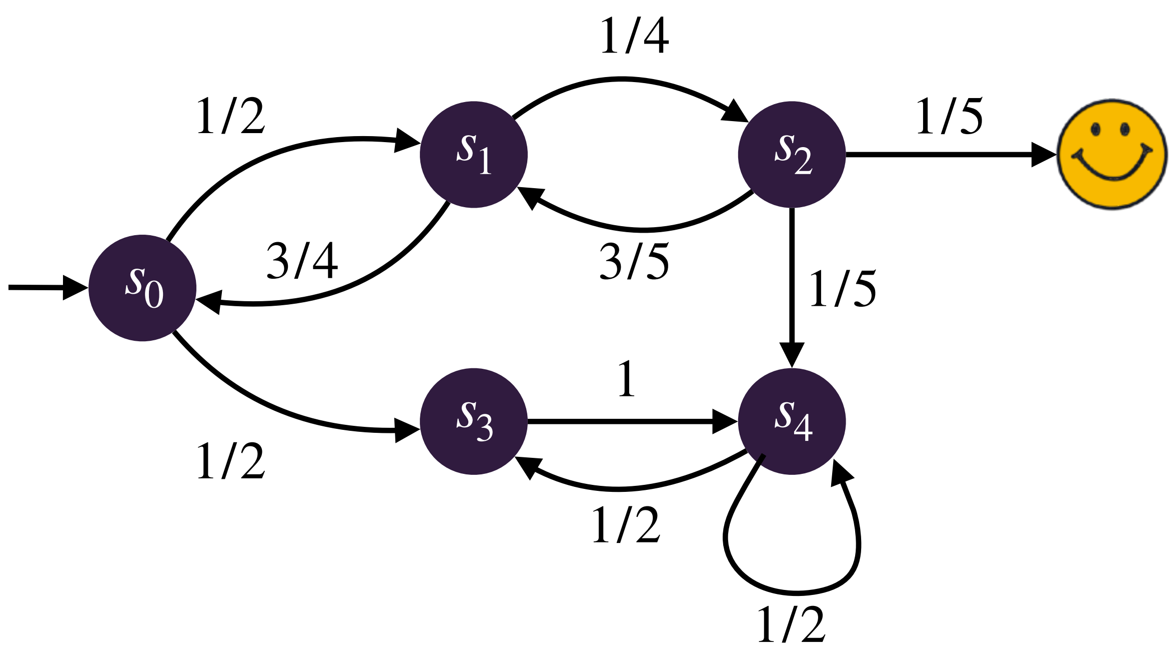

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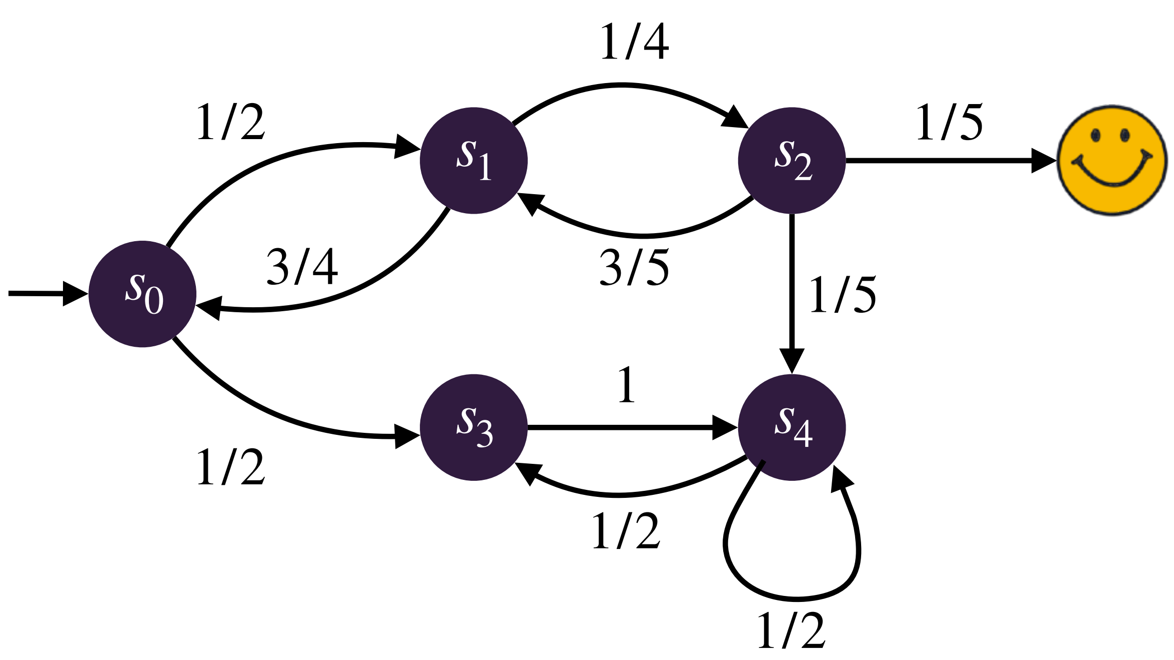

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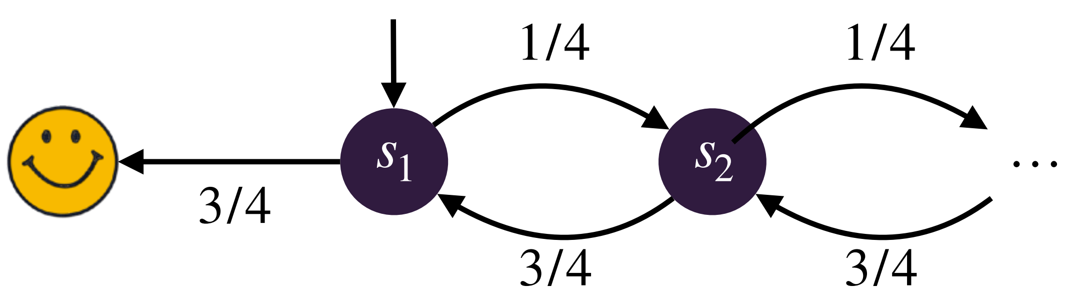

Countable Markov chain
(random walk of parameter $1/4$)

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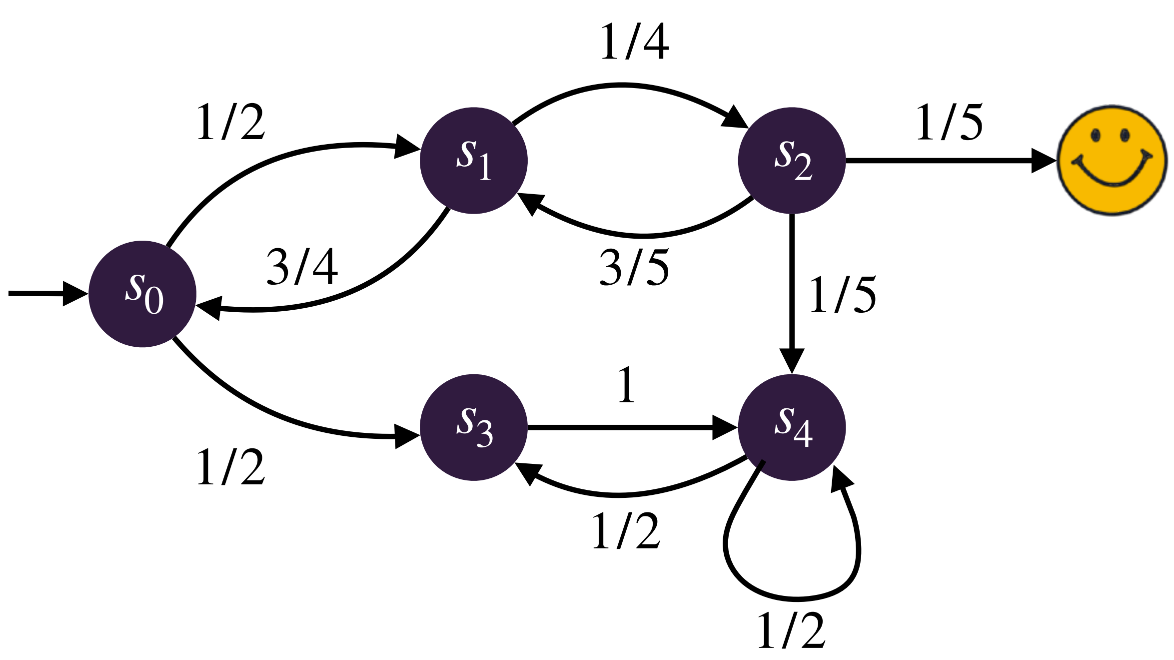
+ effectivity conditions…



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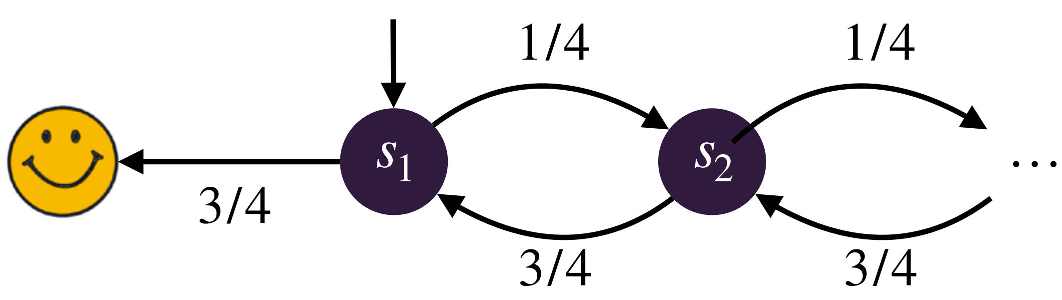

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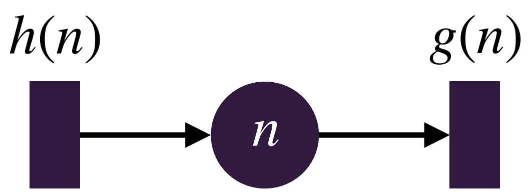
High-level models for (infinite) Markov chains

- Queues



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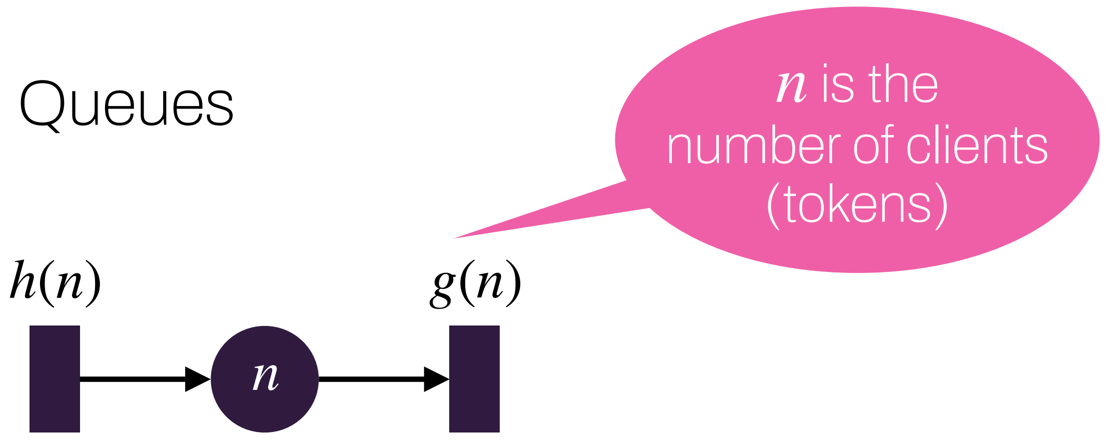

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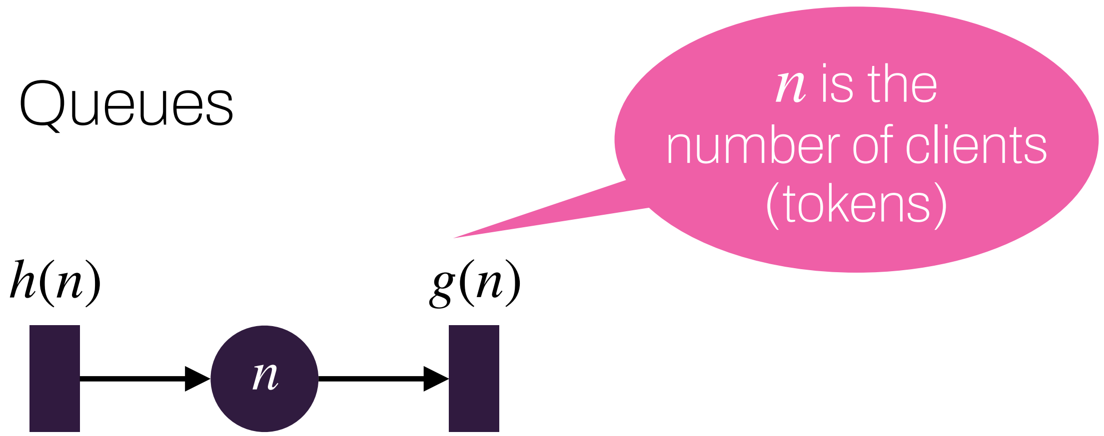

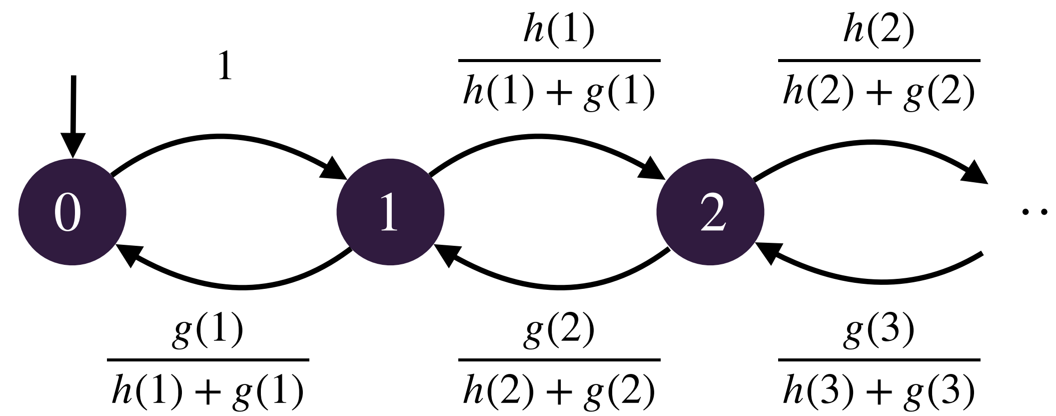

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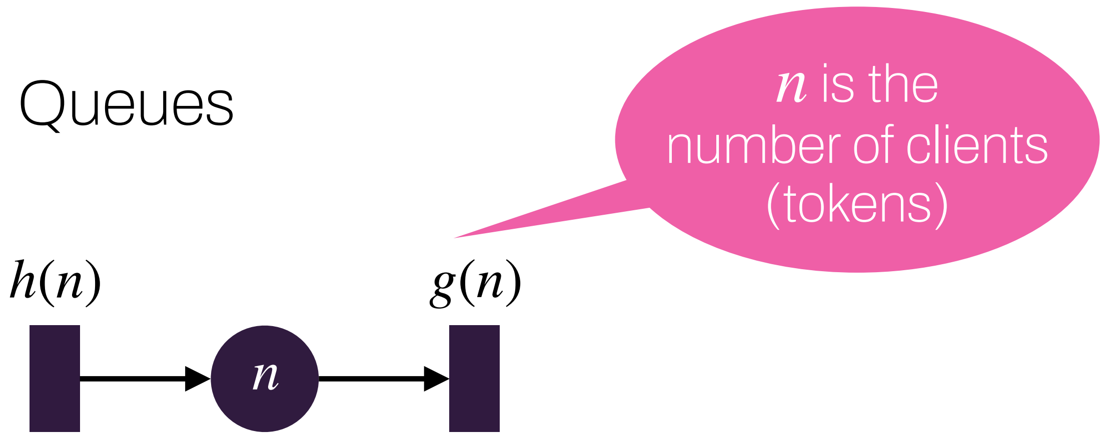

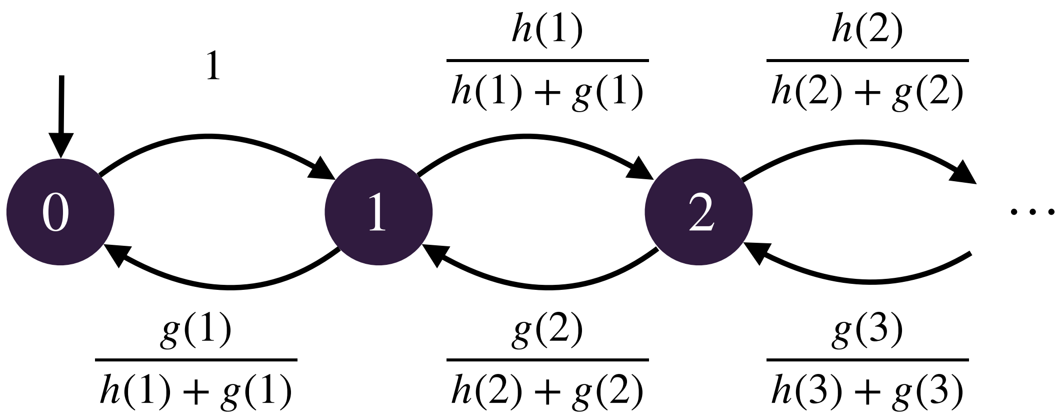

- Probabilistic pushdown automata

$$A \xrightarrow{1} C \quad A \xrightarrow{n} BB \quad B \xrightarrow{5} \varepsilon$$
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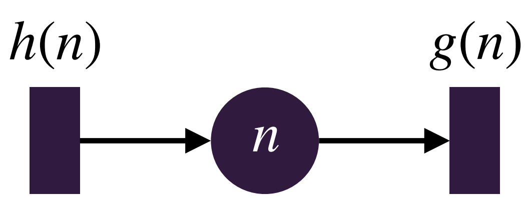
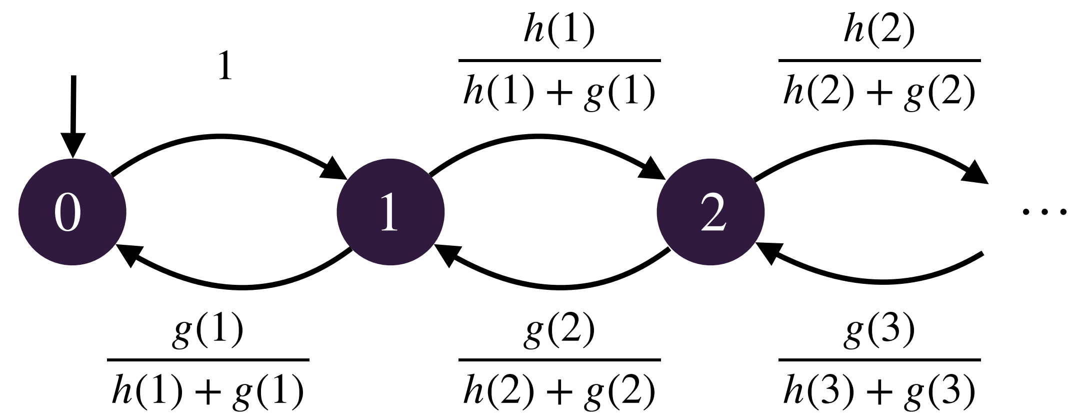
n is the number of clients (tokens)





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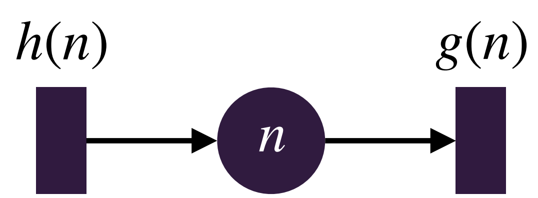

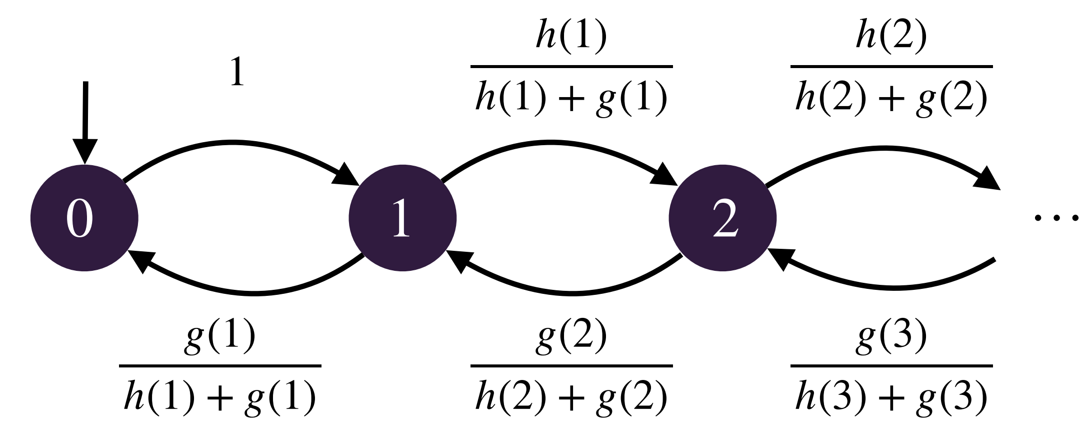

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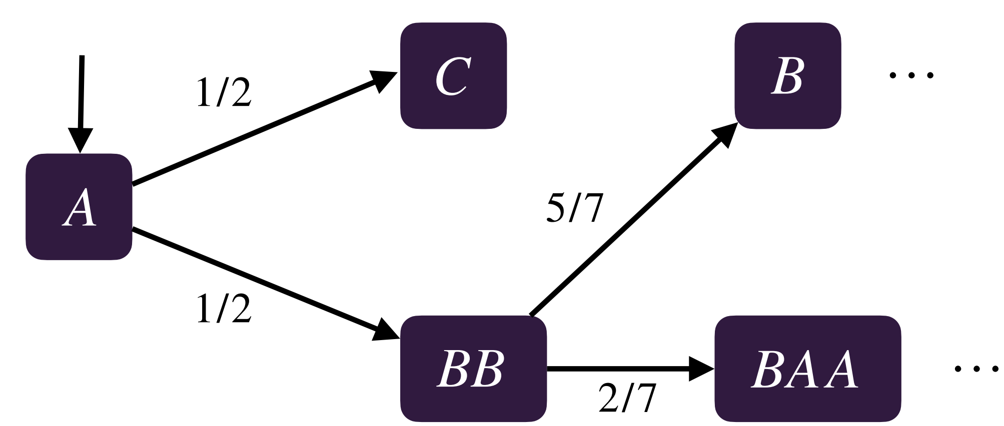
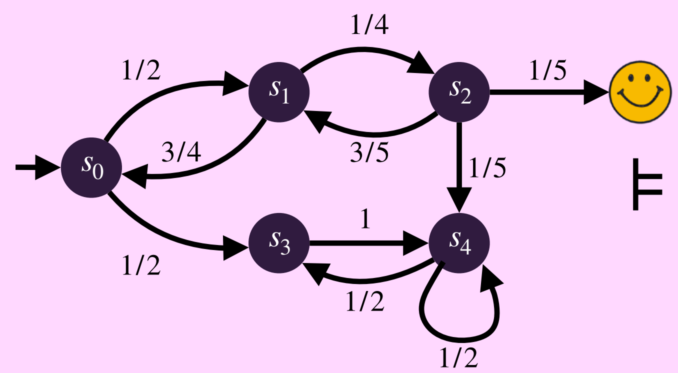


Quantitative analysis of Markov chains

Compute $\mathbb{P}\Big($ 

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[RKNP04] J. Rutten, M. Kwiatkowska, G. Norman, D. Parker. *Mathematical techniques for analyzing concurrent and probabilistic systems (Monograph)*

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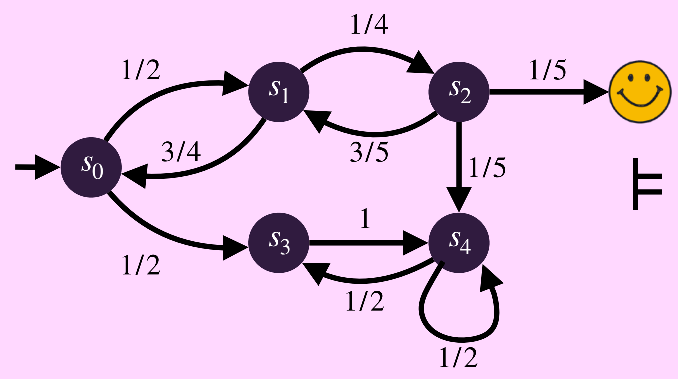
Compute $\mathbb{P}\Big($ 

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Closed-form solution

- Random walk of parameter $p > 1/2$:
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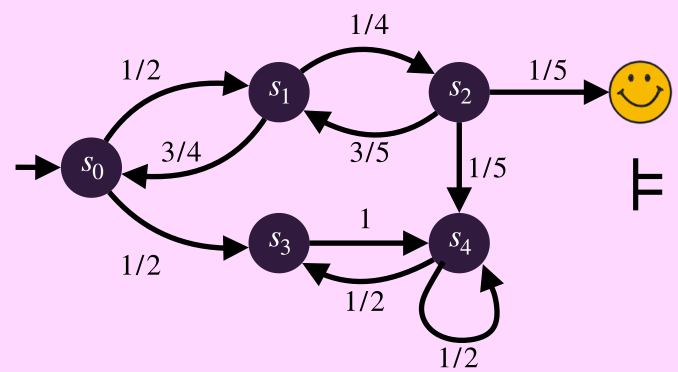
Compute $\mathbb{P}\Big($ 

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Apply a numerical method [RKPN04]

- $$x_s = \begin{cases} 1 & \text{if } s = \text{☺} \\ 0 & \text{if } s \not\models \exists \mathbf{F}\text{☺} \\ \sum_t \mathbb{P}(s \to t) \cdot x_t & \text{otherwise} \end{cases}$$
- $\mathbb{P}_{s_0}(\mathbf{F}$ ☺$) = 1/19$
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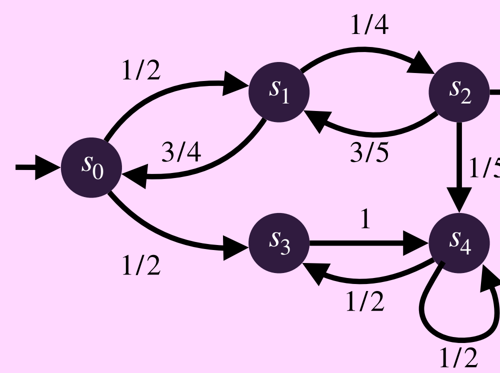
Compute $\mathbb{P}\Big($ 

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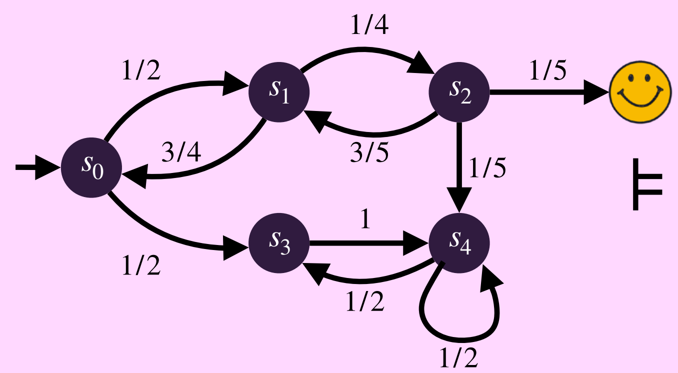
Compute $\mathbb{P}\Big($ 

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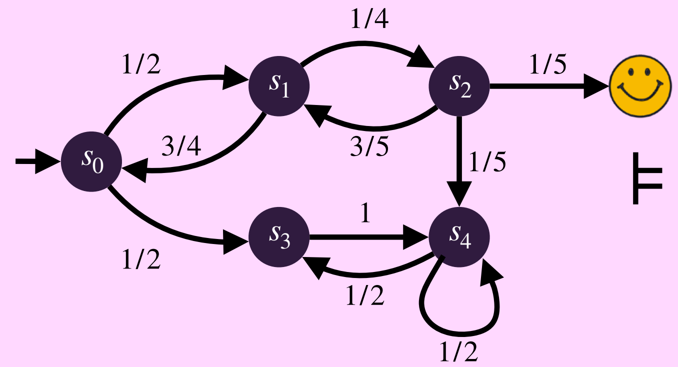
Compute $\mathbb{P}\Big($  $\models \mathbf{F}$ 😊 $\Big)$

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- Specific approaches for **decisive** Markov chains

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Decisiveness

☹️ $= \{s \in S \mid s \not\models \exists \mathbf{F}$ 🙂$\}$

Decisiveness

A DTMC $\mathscr{C}$ is decisive from s w.r.t. 🙂 if $\mathbb{P}_s(\mathbf{F}$ 🙂 $\vee\, \mathbf{F}$ ☹️$) = 1$

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)

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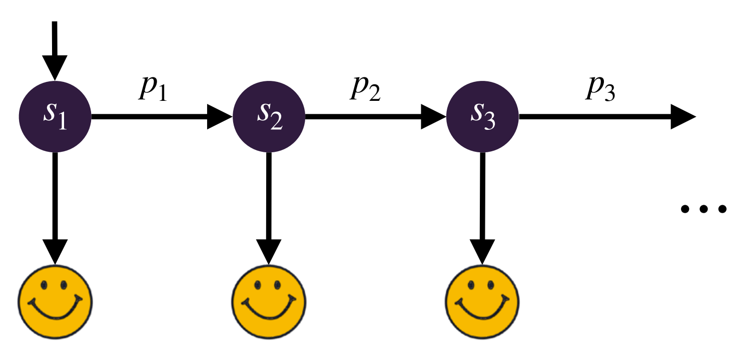

- $\mathbb{P}(\mathbf{G}\neg$ 🙂$) = \prod_{i \geq 1} p_i$
- Decisive iff this product equals 0

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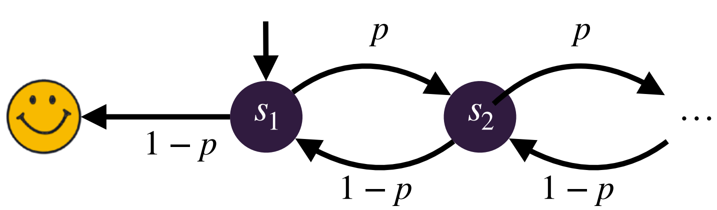

- Recurrent random walk ($p \leq 1/2$): decisive
- Transient random walk ($p > 1/2$): not decisive

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Deciding decisiveness?

Classes where decisiveness can be decided

- Probabilistic pushdown automata with constant weights [ABM07]
- Random walks with polynomial weights [FHY23]
- So-called probabilistic homogeneous one-counter machines with polynomial weights (this extends the model of quasi-birth death processes) [FHY23]

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)
[FHY23] A. Finkel, S. Haddad, L. Yé. About decisiveness of dynamic probabilistic models (CONCUR'23)

Approximation scheme

- Aim: compute probability of $\mathbf{F}$ 🙂
- ☹️ $= \{ s \in S \mid s \not\models \exists \mathbf{F}$ 🙂 $\}$

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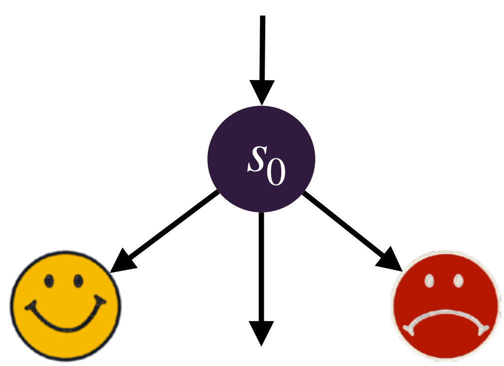

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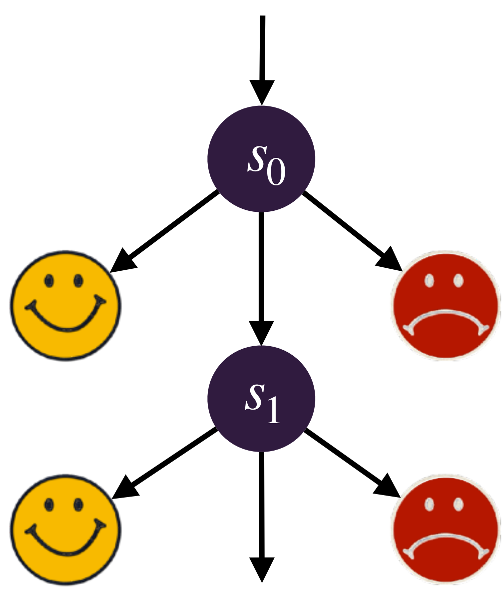

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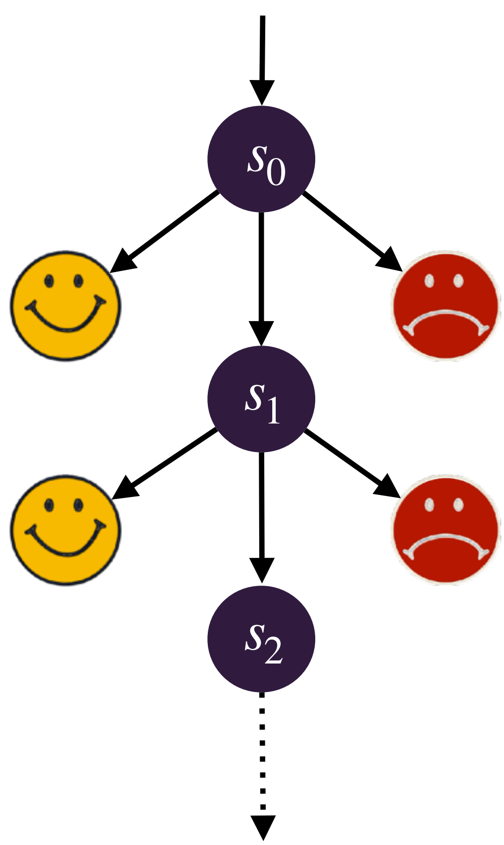

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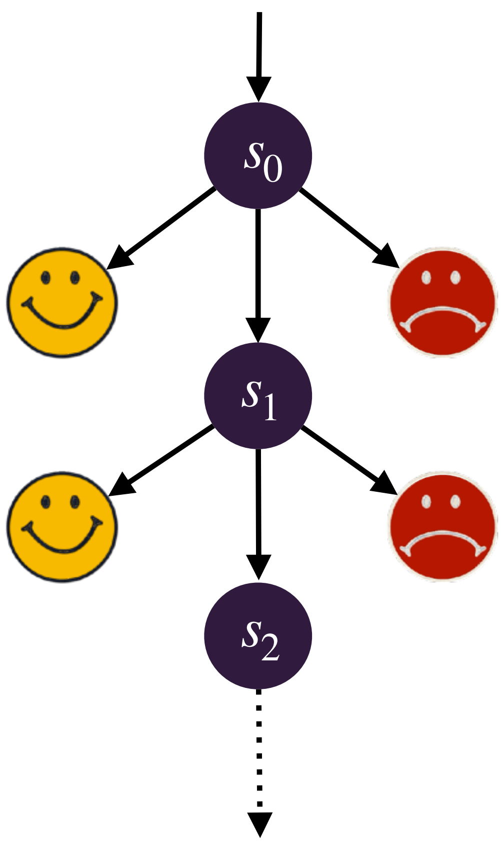

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At the limit: $\mathbb{P}(\mathbf{F}$  $)$ $\quad 1 - \mathbb{P}(\mathbf{F}$  $)$

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- Aim: compute probability of $\mathbf{F}$ 🙂
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The approximation scheme converges iff $\mathscr{C}$ is decisive from s_0 w.r.t. 🙂

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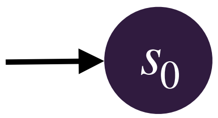
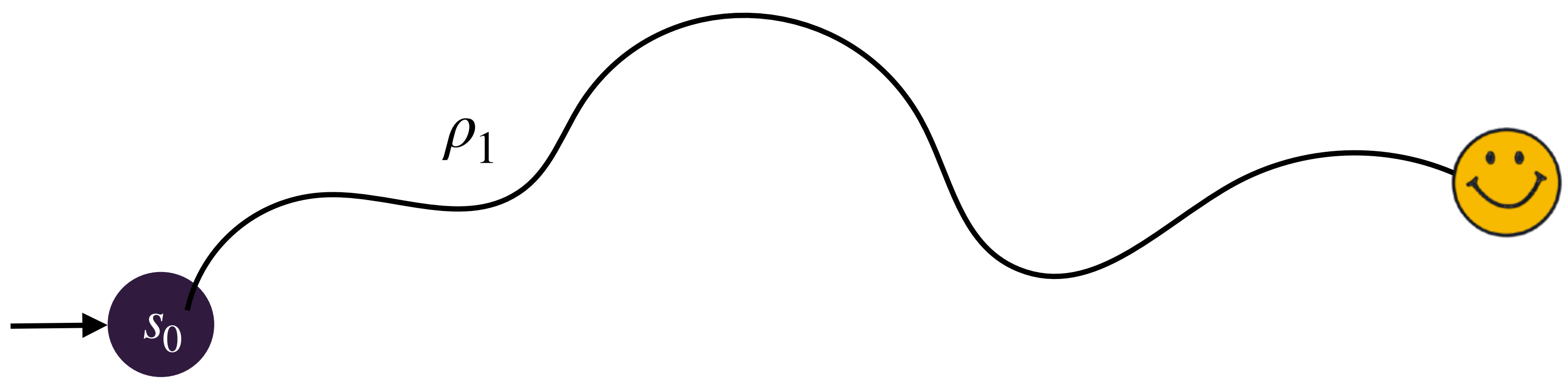
Statistical model-checking

Sample N paths



Statistical model-checking

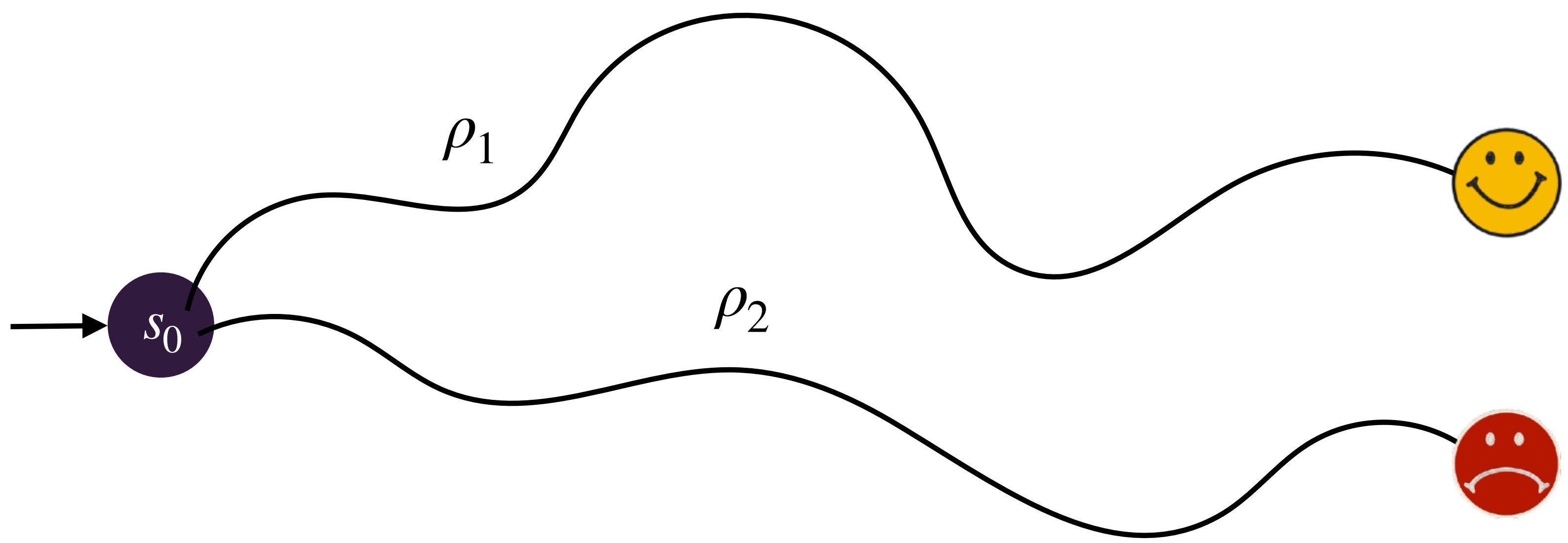
Sample N paths



$n_1 = 1$

Statistical model-checking

Sample N paths

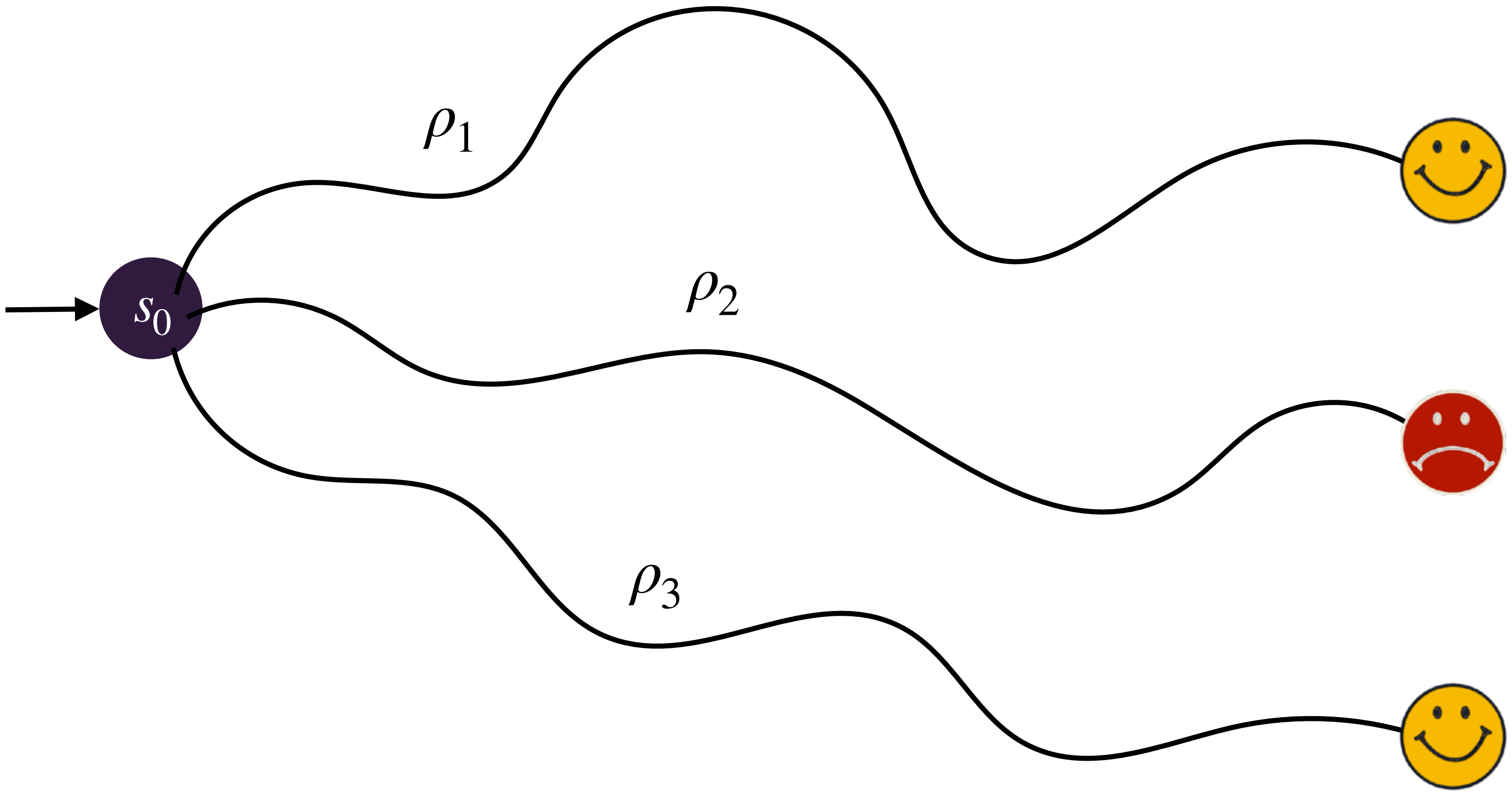


$n_1 = 1$

$n_2 = n_1$

Statistical model-checking

Sample N paths



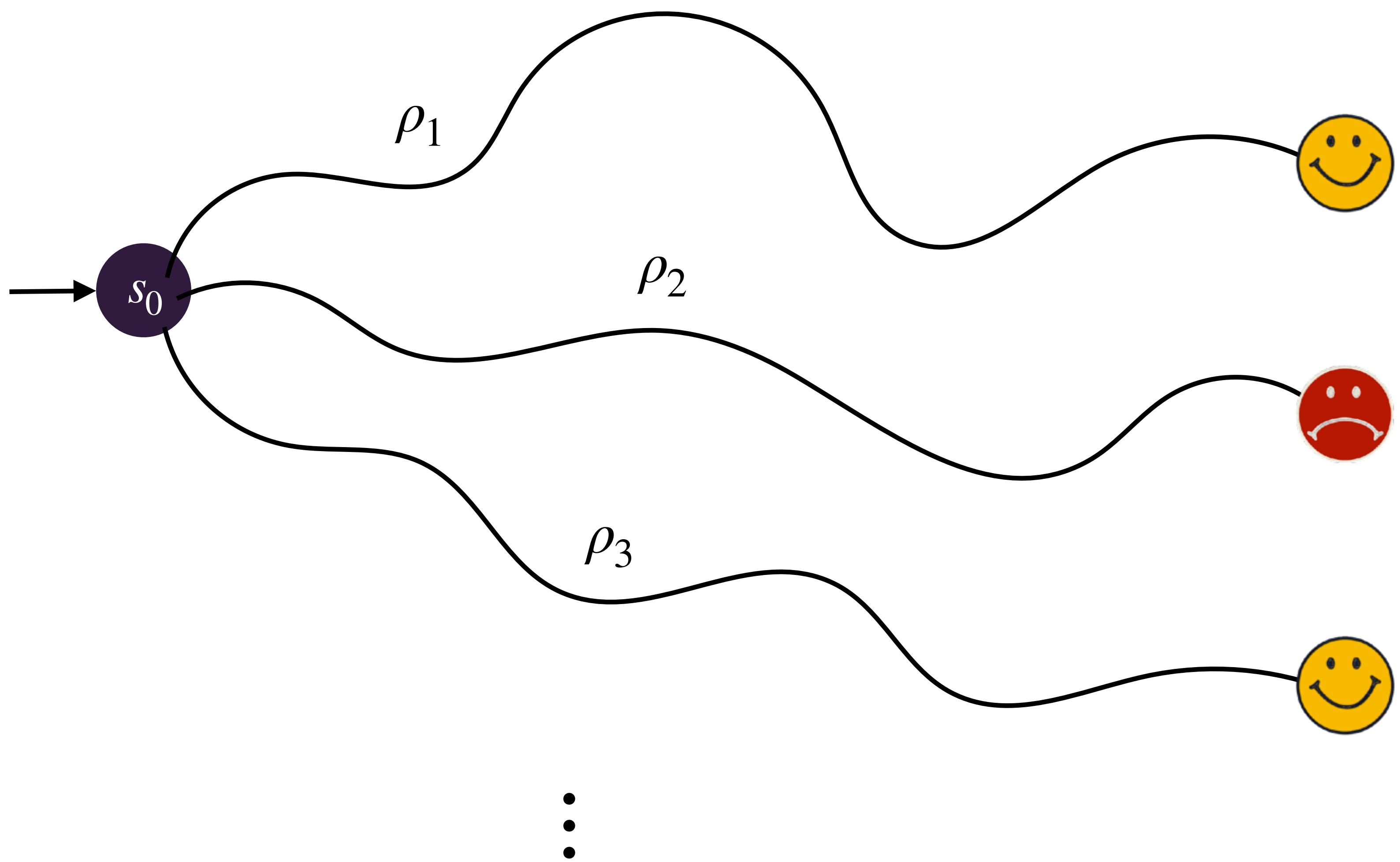
$n_1 = 1$

$n_2 = n_1$

$n_3 = n_2 + 1$

Statistical model-checking

Sample N paths



$n_1 = 1$

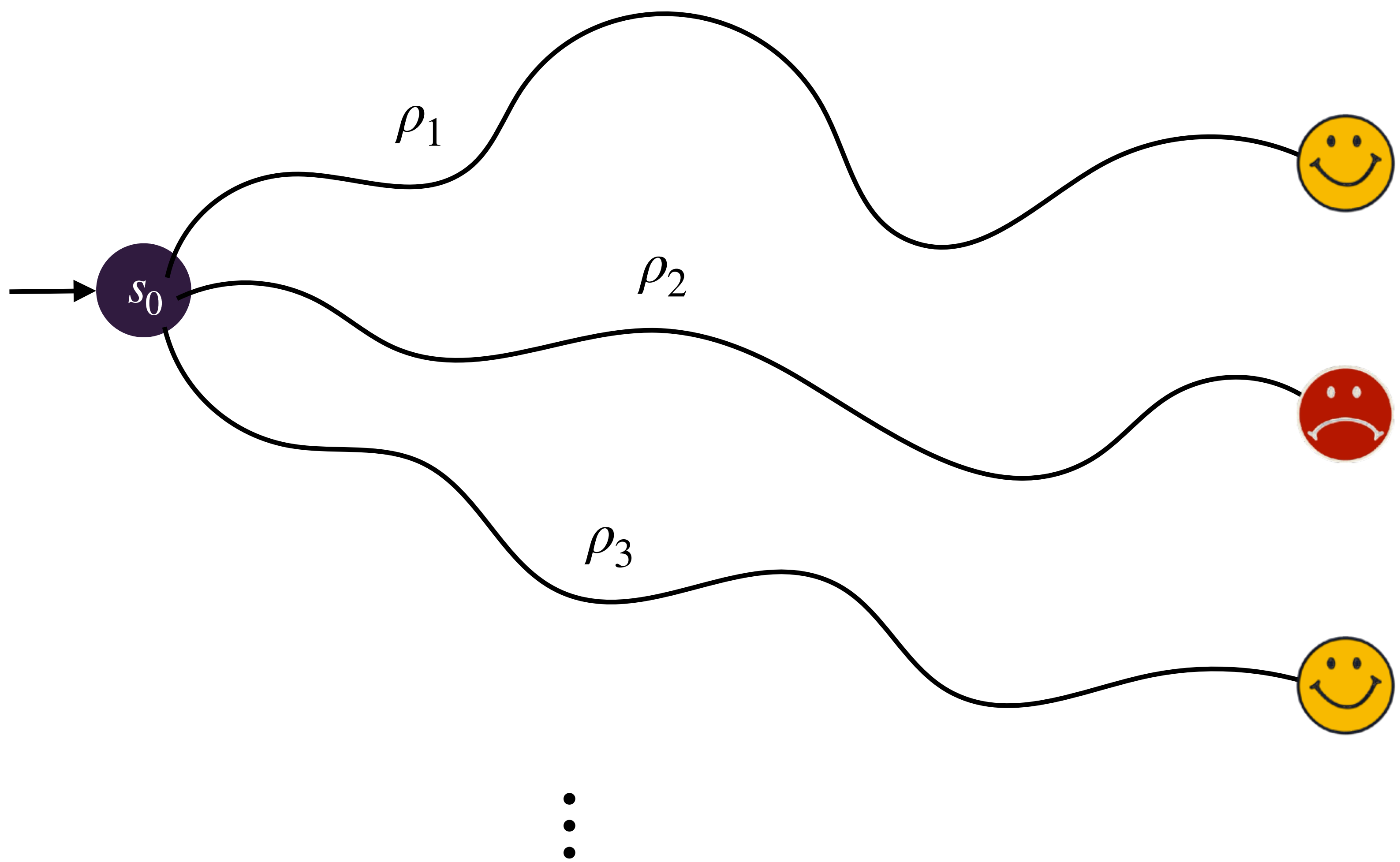
$n_2 = n_1$

$n_3 = n_2 + 1$

⋮

Statistical model-checking

Sample N paths



$n_1 = 1$

$n_2 = n_1$

$n_3 = n_2 + 1$

$\vdots$

Return $\dfrac{n_N}{N}$ + some confidence interval

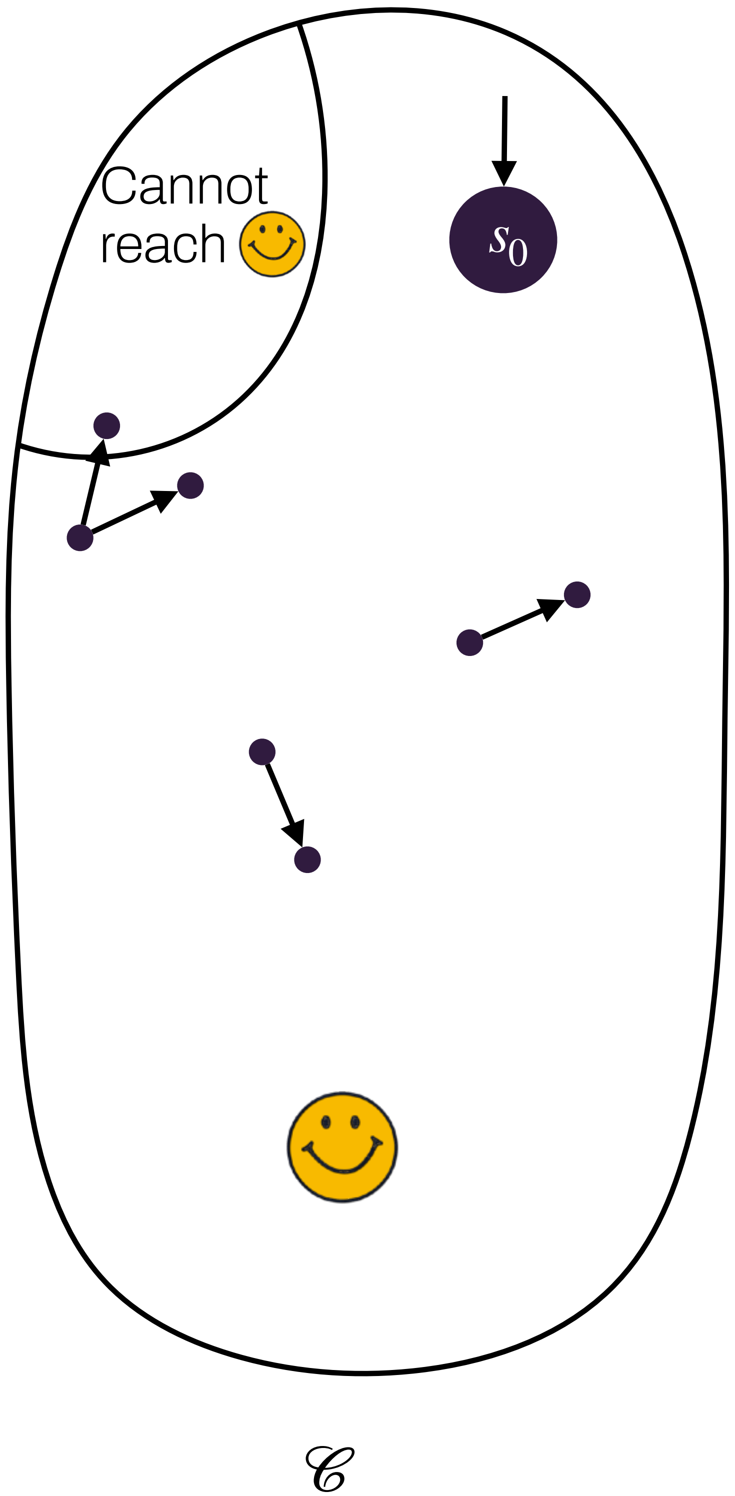
(in the best case)

Termination, efficiency and guarantees

Termination (To our knowledge, never expressed like this)

A sampled path starting at s_0 almost-surely hits  or 
iff
$\mathscr{C}$ is decisive from s_0 w.r.t. 

Decisiveness *vs* recurrence



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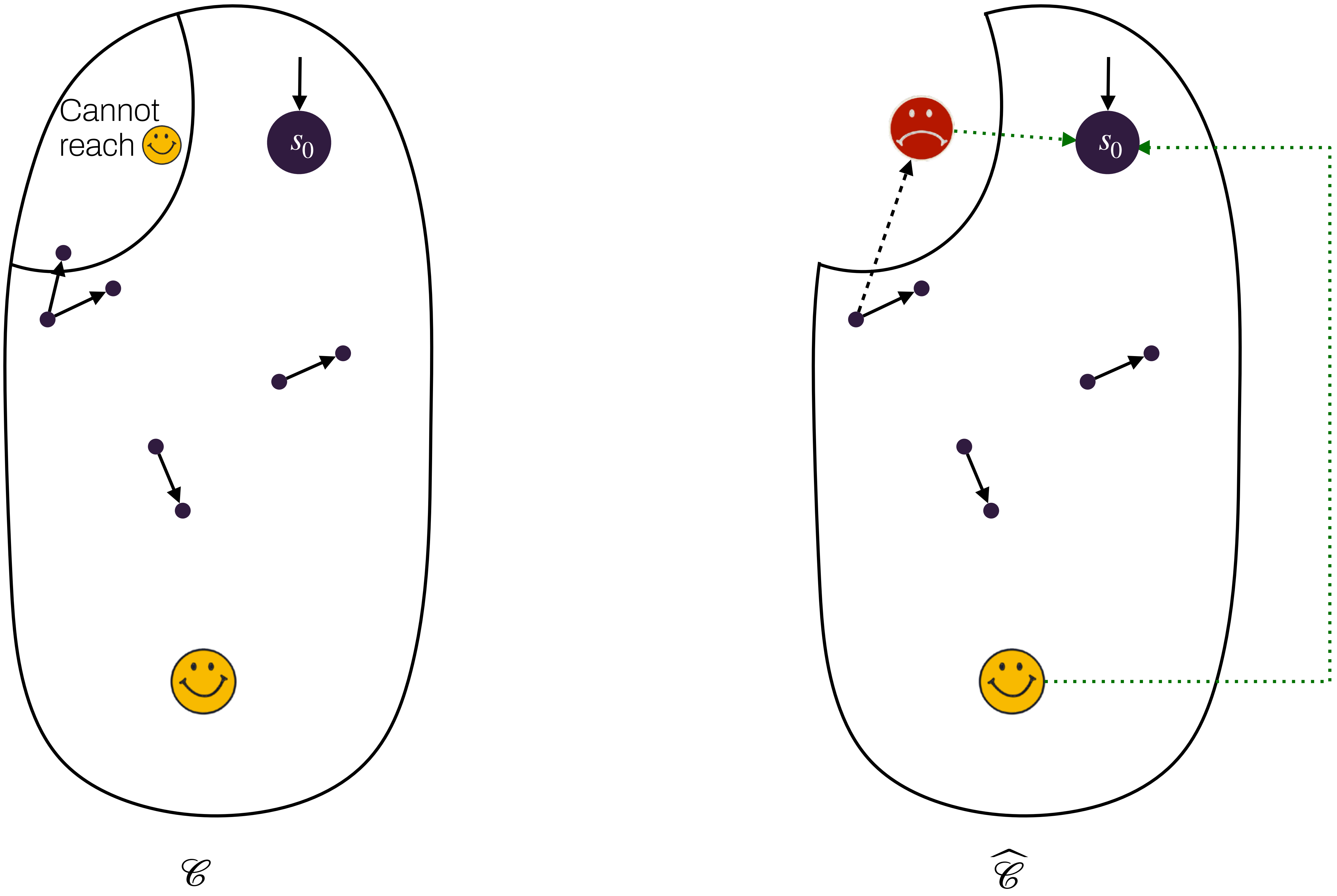
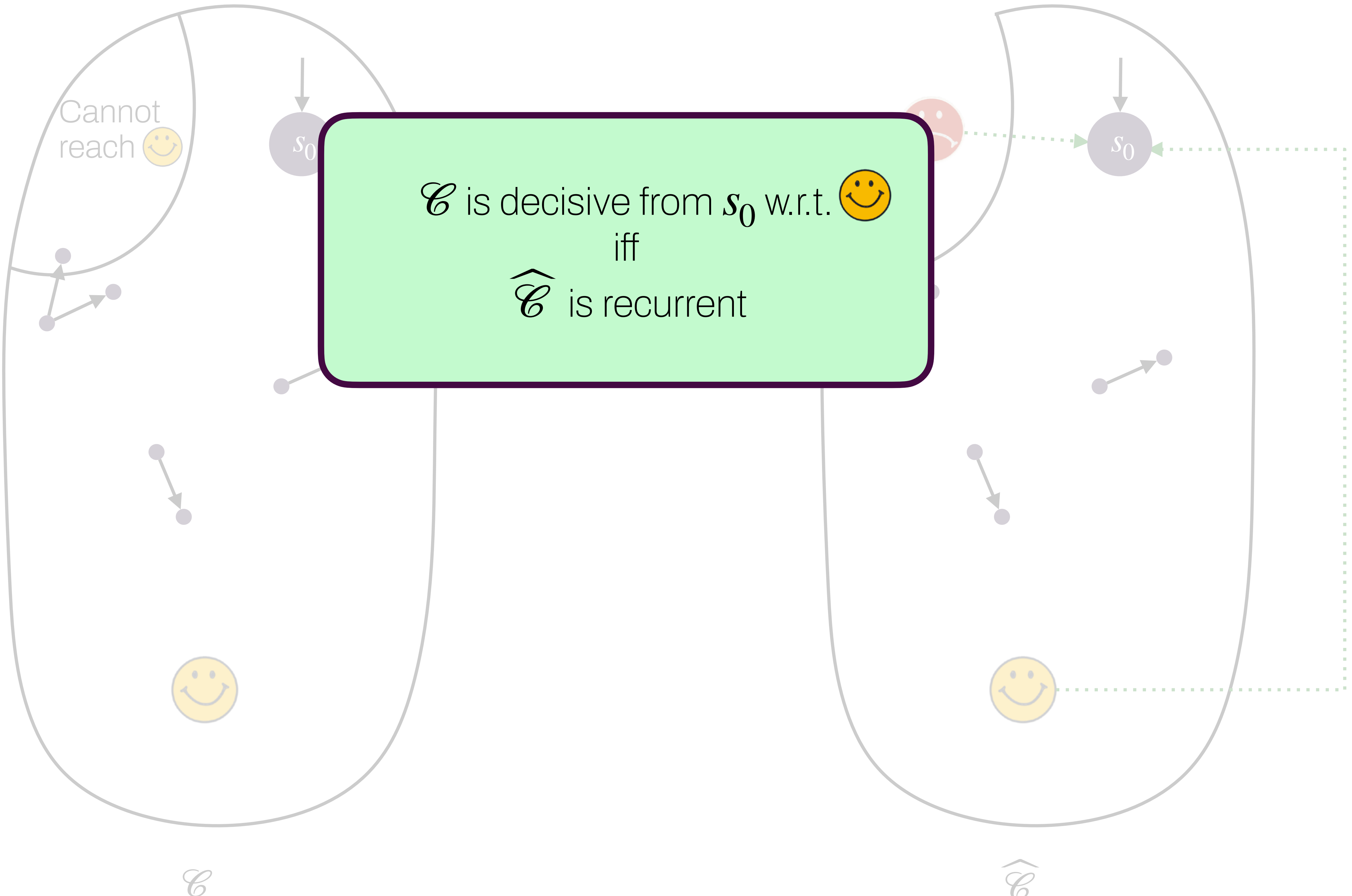
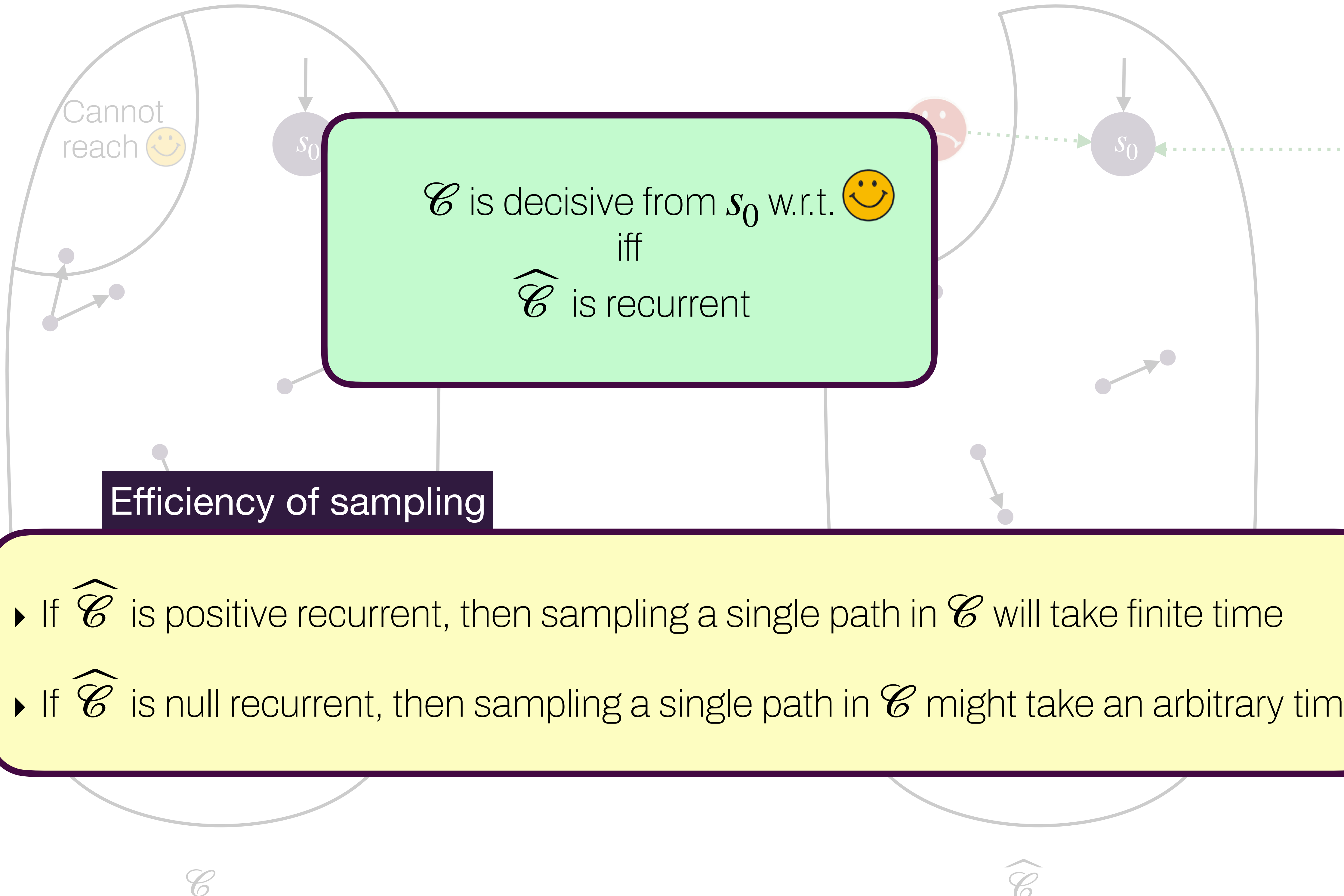


Decisiveness *vs* recurrence



$\mathscr{C}$ is decisive from s_0 w.r.t. 😊
iff
$\widehat{\mathscr{C}}$ is recurrent

Decisiveness *vs* recurrence



$\mathscr{C}$ is decisive from s_0 w.r.t. 😊

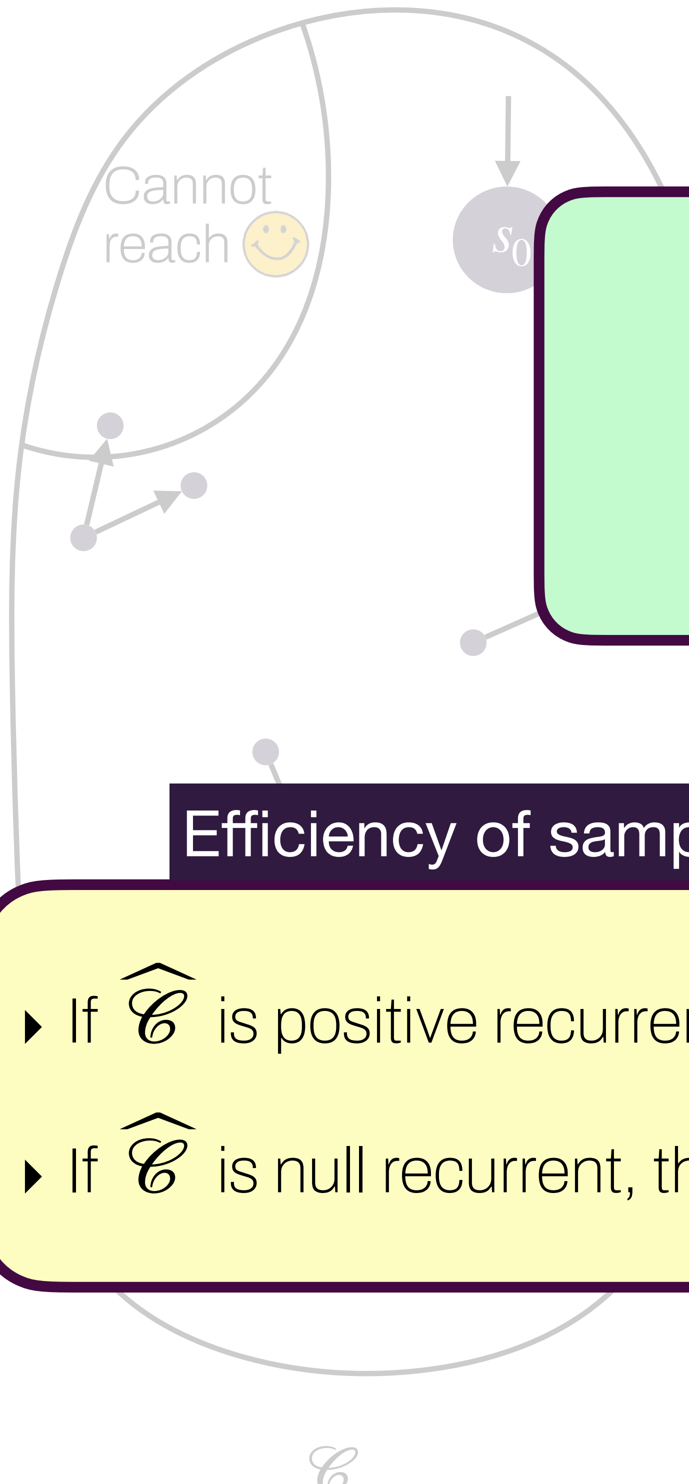
iff

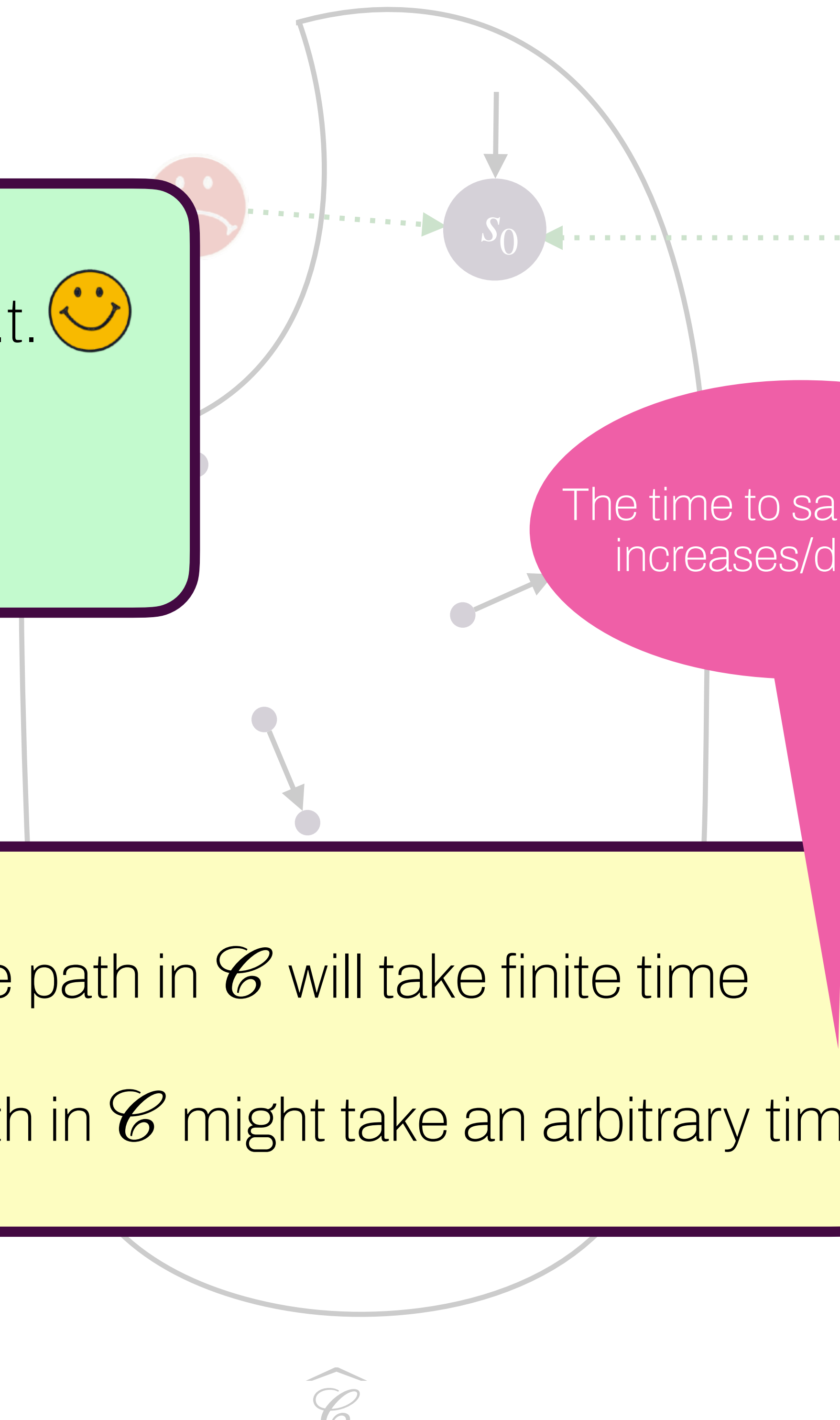
$\widehat{\mathscr{C}}$ is recurrent

Efficiency of sampling

- If $\widehat{\mathscr{C}}$ is positive recurrent, then sampling a single path in $\mathscr{C}$ will take finite time
- If $\widehat{\mathscr{C}}$ is null recurrent, then sampling a single path in $\mathscr{C}$ might take an arbitrary time

Decisiveness vs recurrence





$\mathscr{C}$ is decisive from s_0 w.r.t. 🙂
iff
$\widehat{\mathscr{C}}$ is recurrent

The time to sample even increases/diverges!

Efficiency of sampling

- If $\widehat{\mathscr{C}}$ is positive recurrent, then sampling a single path in $\mathscr{C}$ will take finite time
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+ efficiency if finite return time
(« $\widehat{\mathscr{C}}$ is positive recurrent »)

Termination, efficiency and guarantees

Termination

(To our knowledge, never expressed like this)

A sampled path starting at s_0 almost-surely hits 🙂 or ☹️
iff
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+ efficiency if finite return time
(« $\widehat{\mathscr{C}}$ is positive recurrent »)

Guarantees: Hoeffding's inequalities

Let $\varepsilon, \delta > 0$, let $N \geq \frac{8}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

$$\mathbb{P}\left(\left| \frac{n_N}{N} - \mathbb{P}(\mathbf{F}\ 🙂\ ) \right| \geq \frac{\varepsilon}{2} \right) \leq \delta$$

Termination, efficiency and guarantees

Termination

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Empirical average

$$\mathbb{P}\left(\left|\frac{n_N}{N} - \mathbb{P}(\mathbf{F}\,🙂)\right| \geq \frac{\varepsilon}{2}\right) \leq \delta$$

Termination, efficiency and guarantees

Termination

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Empirical average

Precision

Termination, efficiency and guarantees

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Empirical average

Confidence level

Precision

Termination, efficiency and guarantees

Termination

(To our knowledge, never expressed like this)

A sampled path starting at s_0 almost-surely hits 😊 or ☹
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$\mathscr{C}$ is decisive from s_0 w.r.t. 😊

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Guarantees: Hoeffding's inequalities

Let $\varepsilon, \delta > 0$, let $N \geq \frac{8}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

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Empirical average

Confidence level

Precision

$\left[\frac{n_N}{N} - \frac{\varepsilon}{2}; \frac{n_N}{N} + \frac{\varepsilon}{2}\right]$: confidence interval

Termination, efficiency and guarantees

Termination

(To our knowledge, never expressed like this)

A sampled path starting at s_0 almost-surely hits ☺ or ☹
iff
$\mathscr{C}$ is decisive from s_0 w.r.t. ☺

+ efficiency if finite return time
(« $\mathscr{C}$ is positive recurrent »)

Guarantees: Hoeffding's inequalities

Let $\varepsilon, \delta > 0$ s.t. $N \geq \dfrac{8B^2}{\varepsilon^2} \log\left(\dfrac{2}{\delta}\right)$. Then:

B bound on the function

Empirical estimation

$$\mathbb{P}\left(\left|\frac{f_N}{N} - \mathbb{E}(f_{L,☺})\right| \geq \frac{\varepsilon}{2}\right) \leq \delta$$

$\left[\dfrac{f_N}{N} - \dfrac{\varepsilon}{2}; \dfrac{f_N}{N} + \dfrac{\varepsilon}{2}\right]$: confidence interval

Value given by L for paths that stop at ☺

What can we do for
non-decisive Markov chains??

Importance sampling [KH51]

[KH51] H. Kahn, T. E. Harris. Estimation of particle transmission by random sampling (National Bureau of Standards applied mathematics series, 1951)
[Bar14] B. Barbot. Acceleration for statistical model checking (PhD thesis)
[BHP12] B. Barbot, S. Haddad, C. Picaronny. Coupling and Importance Sampling for Statistical Model Checking (TACAS'12)

Importance sampling [KH51]

- Originally used for rare events

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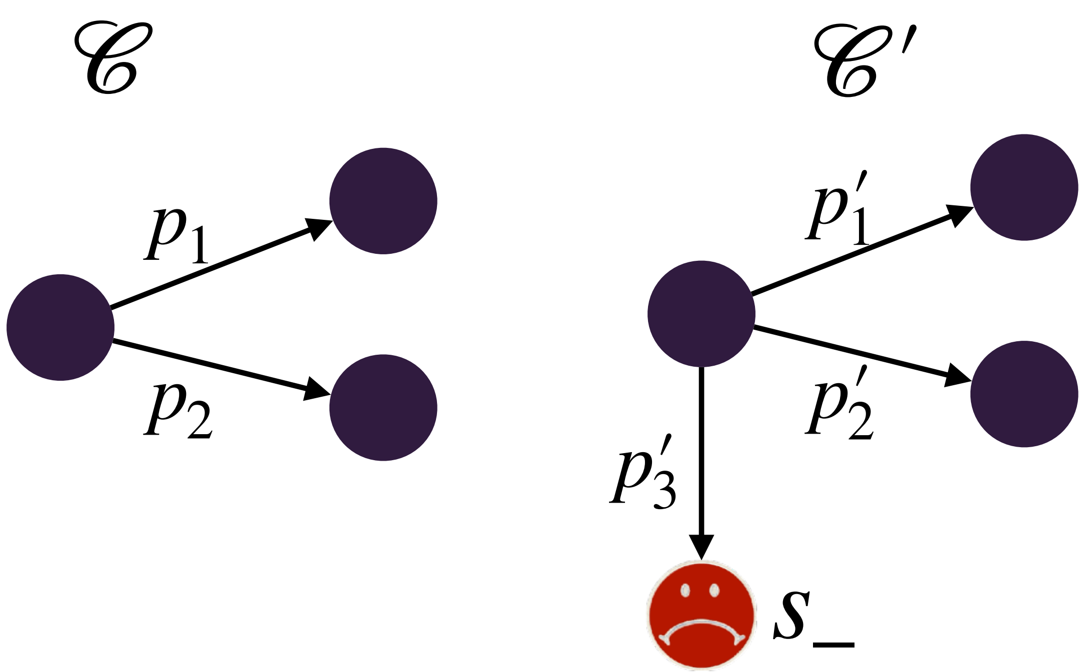
Importance sampling [KH51]

- Analyze a biased Markov chain $\mathscr{C}'$



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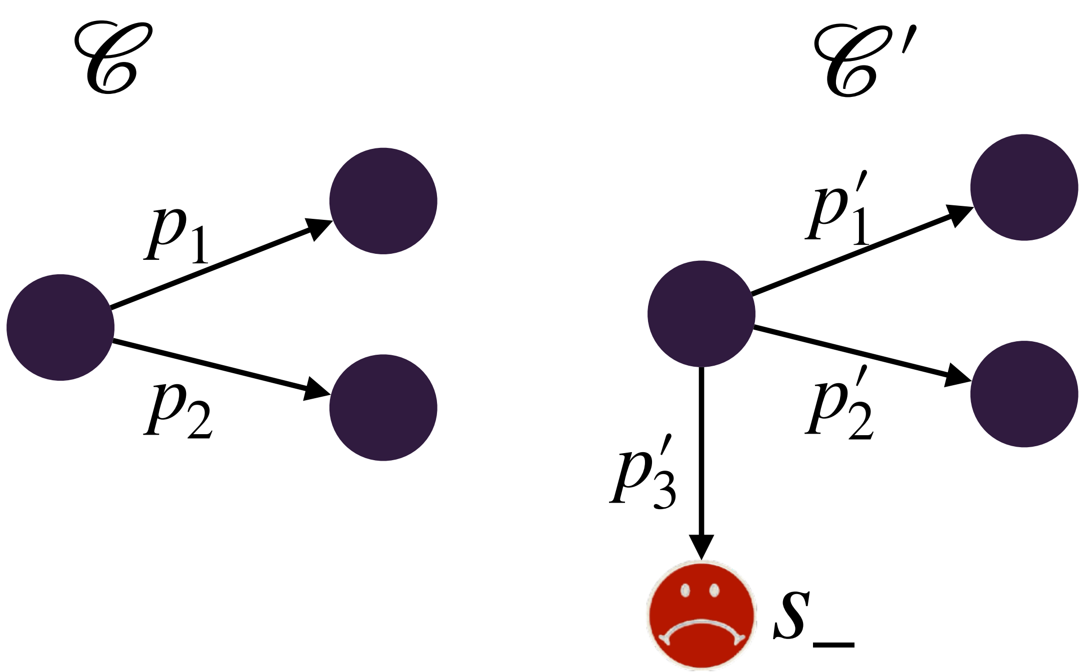

Correct the bias

$$\gamma(\rho) = \begin{cases} \frac{P(\rho)}{P'(\rho)} & \text{if } \rho \text{ ends in } ☺ \\ 0 & \text{otherwise} \end{cases}$$

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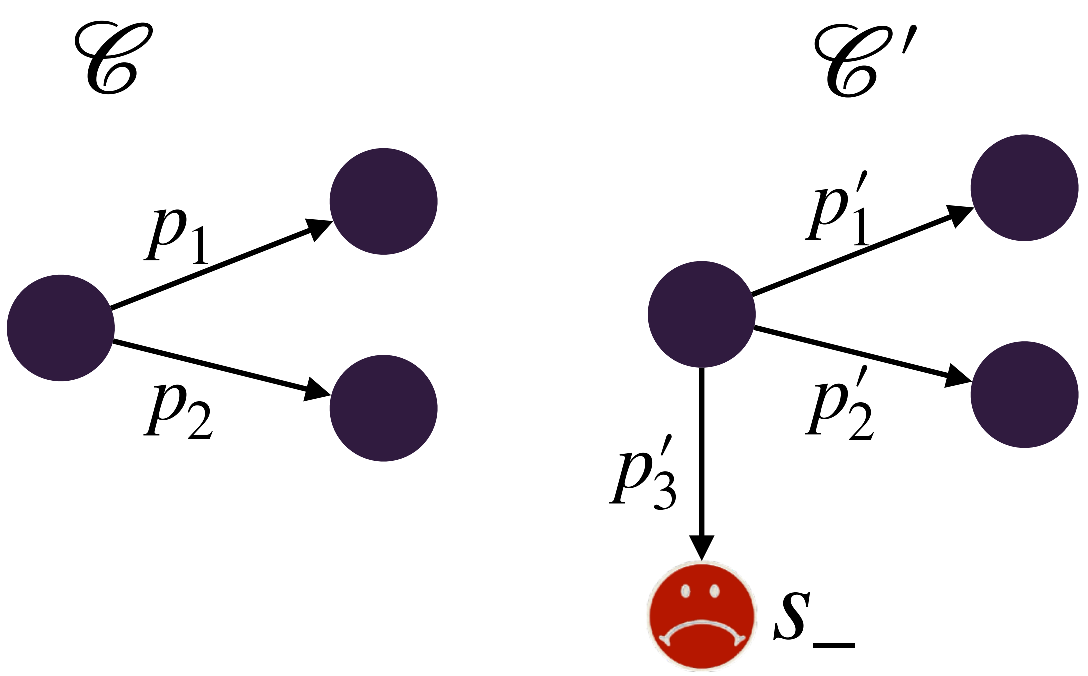

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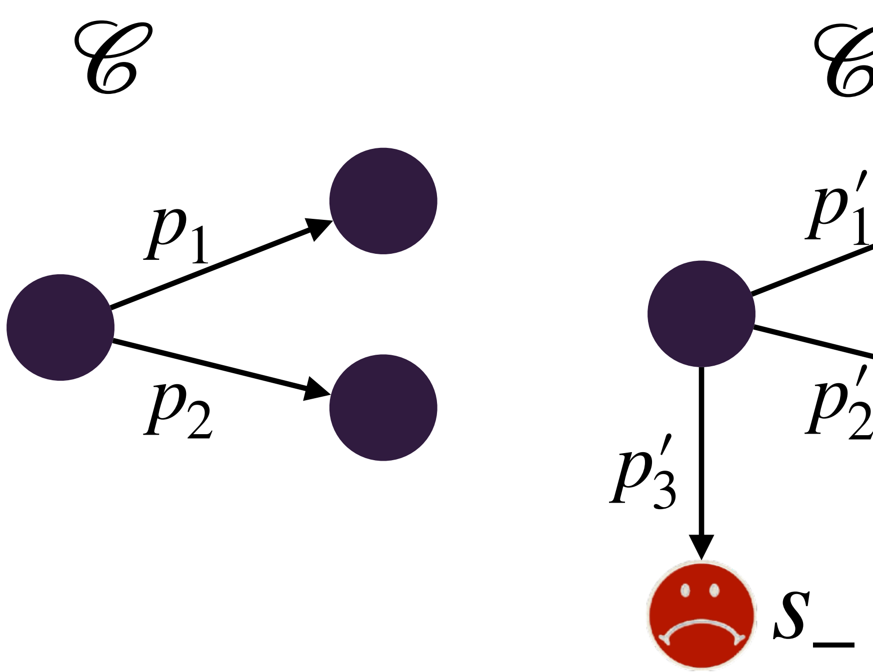

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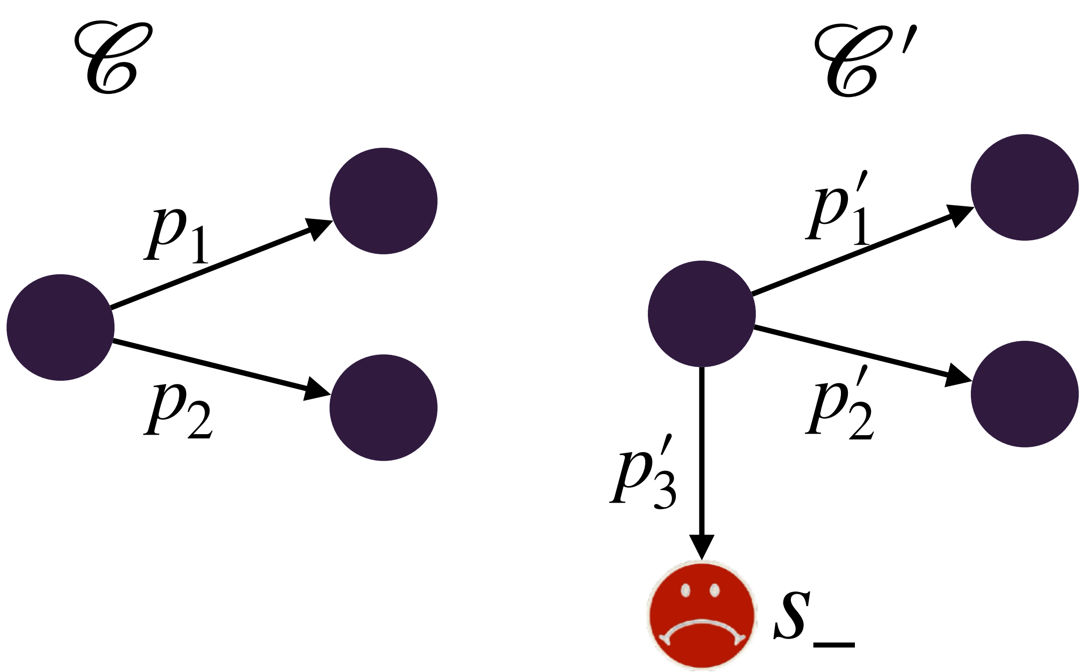

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- Setting giving statistical guarantees [BHP12,Bar14]

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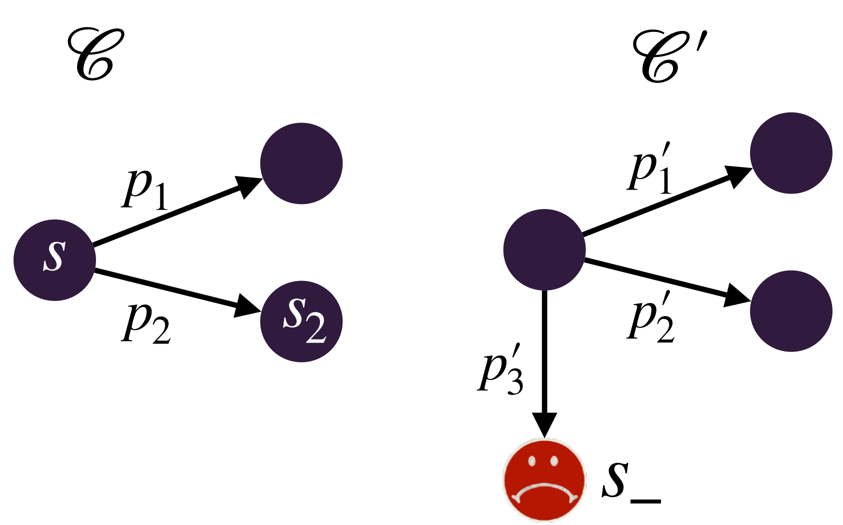
Properties of the biased Markov chain

$$\mathbb{P}_{\mathscr{C}}(\mathbf{F}\ 😊) = \mathbb{E}_{\mathscr{C}'}(\gamma)$$



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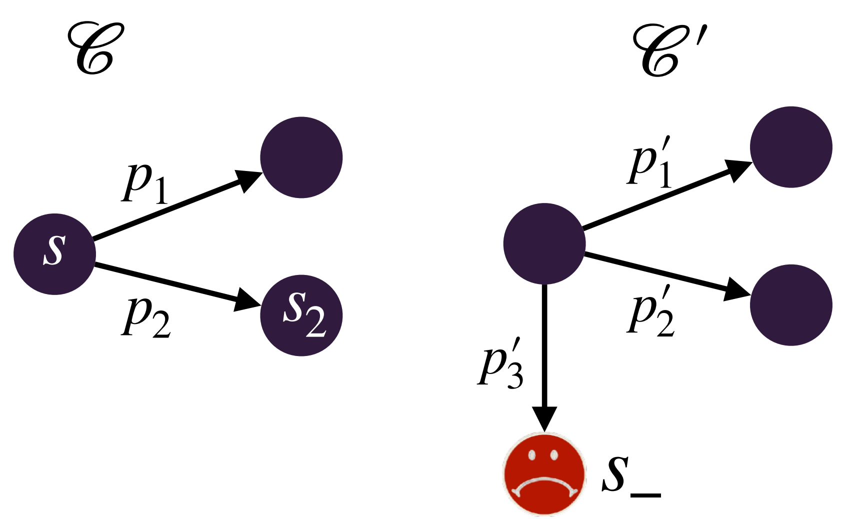

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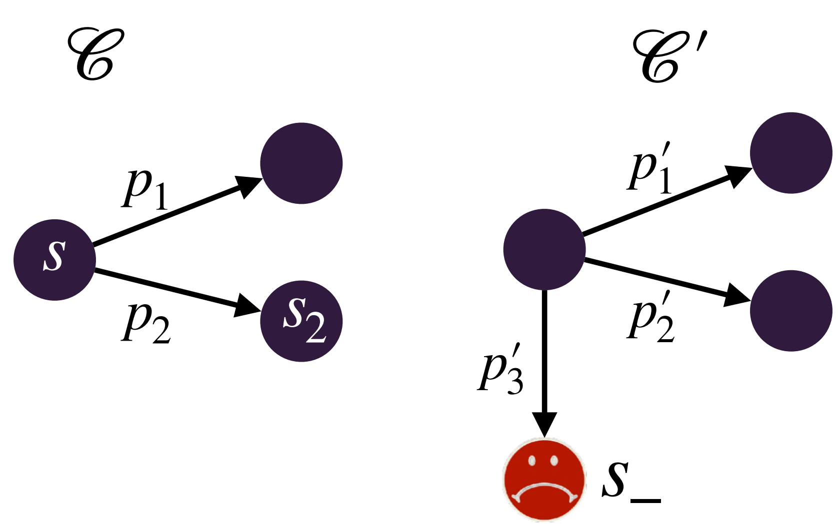

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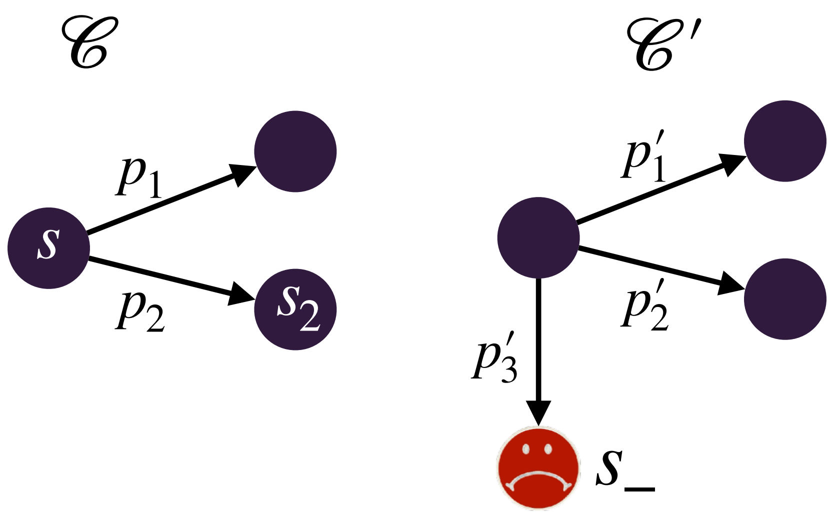

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Properties of the biased Markov chain

Define $\mu(s)$ as $\mathbb{P}^{s}_{\mathscr{C}}(\mathbf{F}$ ☺$)$

$$\mathbb{P}_{\mathscr{C}}(\mathbf{F}\ ☺) = \mathbb{E}_{\mathscr{C}'}(\gamma)$$

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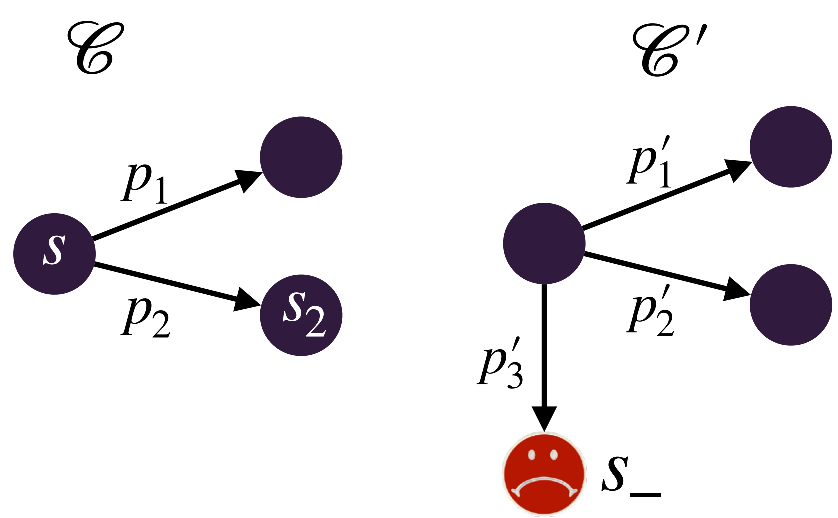

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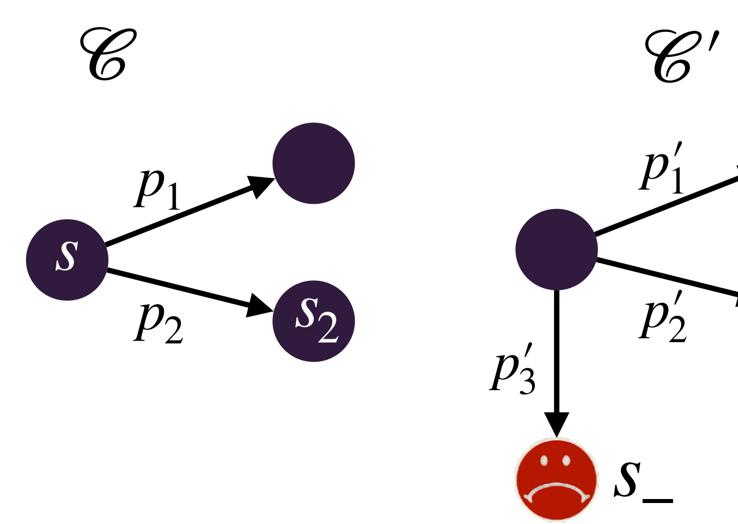
There is a best choice: $p'_i = \dfrac{\mu(s_i)}{\mu(s)} \cdot p_i$

- The r.v. in $\mathscr{C}'$ takes value $\mu(s)$
- One needs to know μ!



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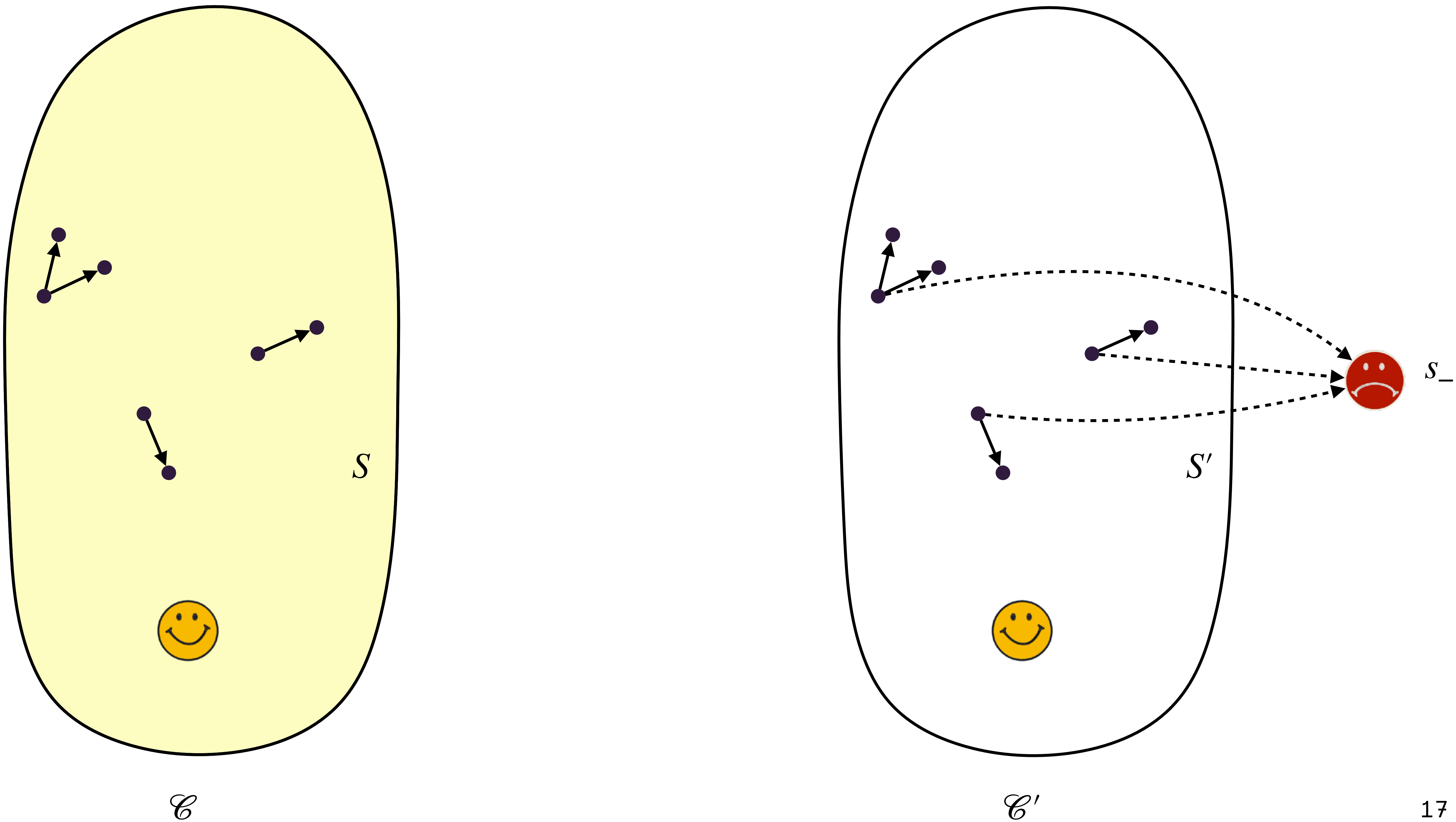
Importance sampling via an abstraction



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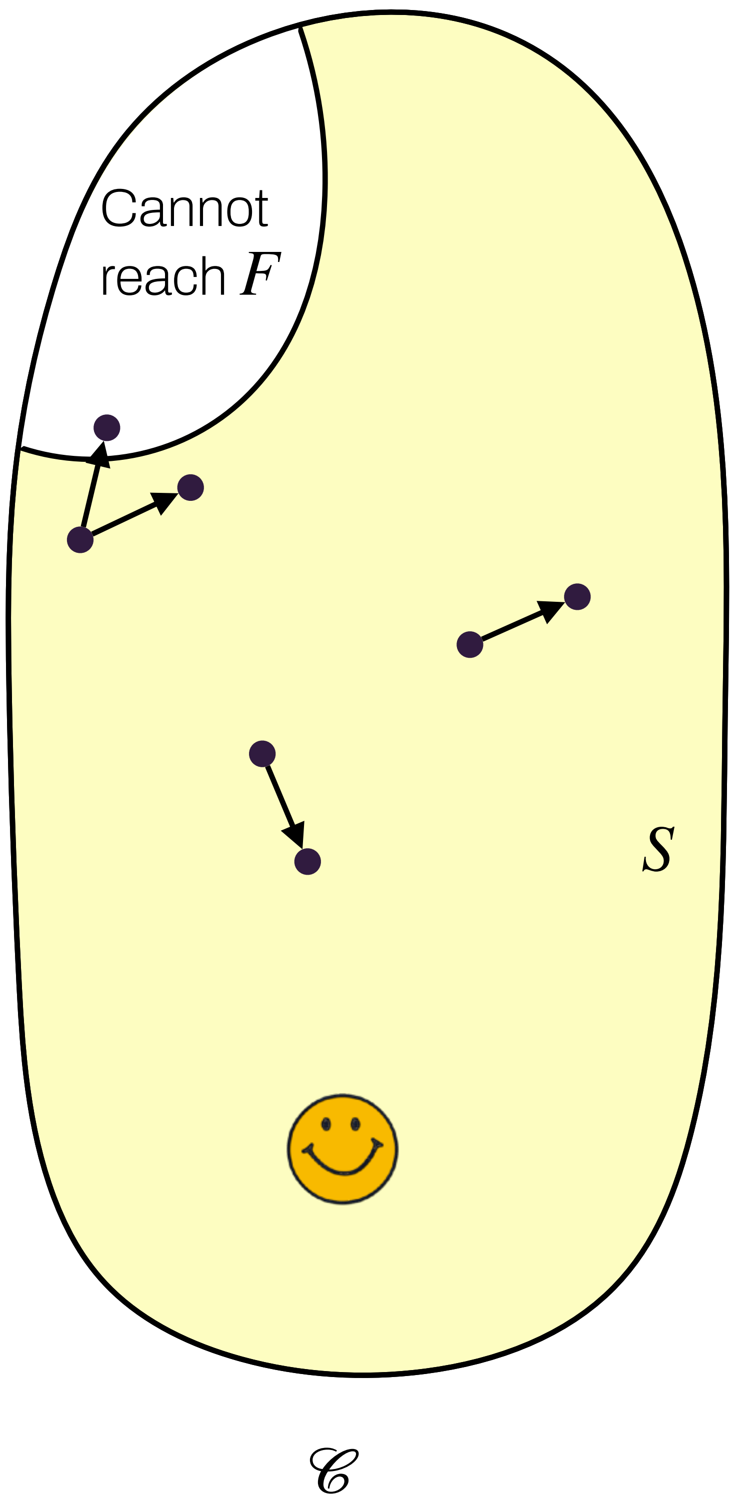

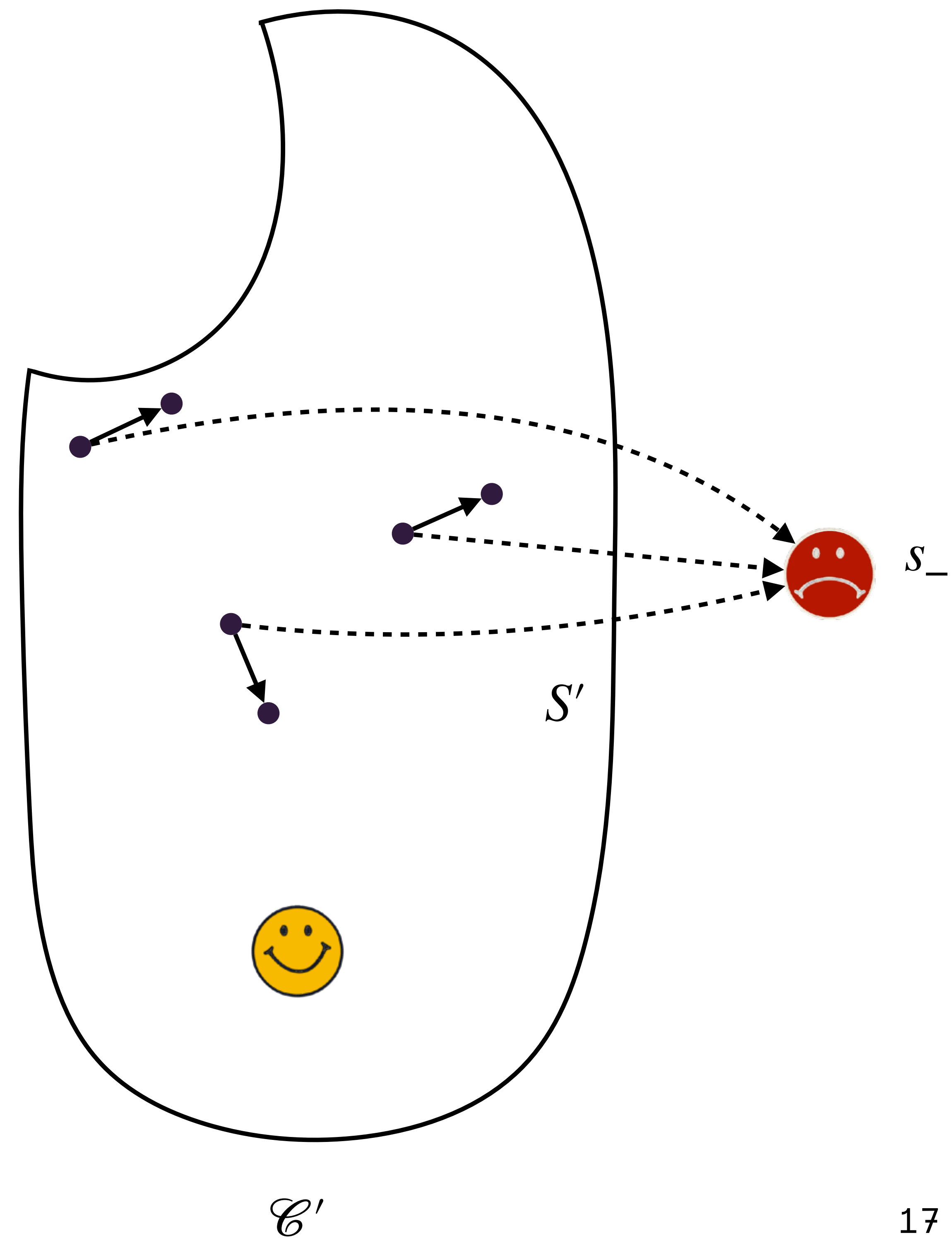

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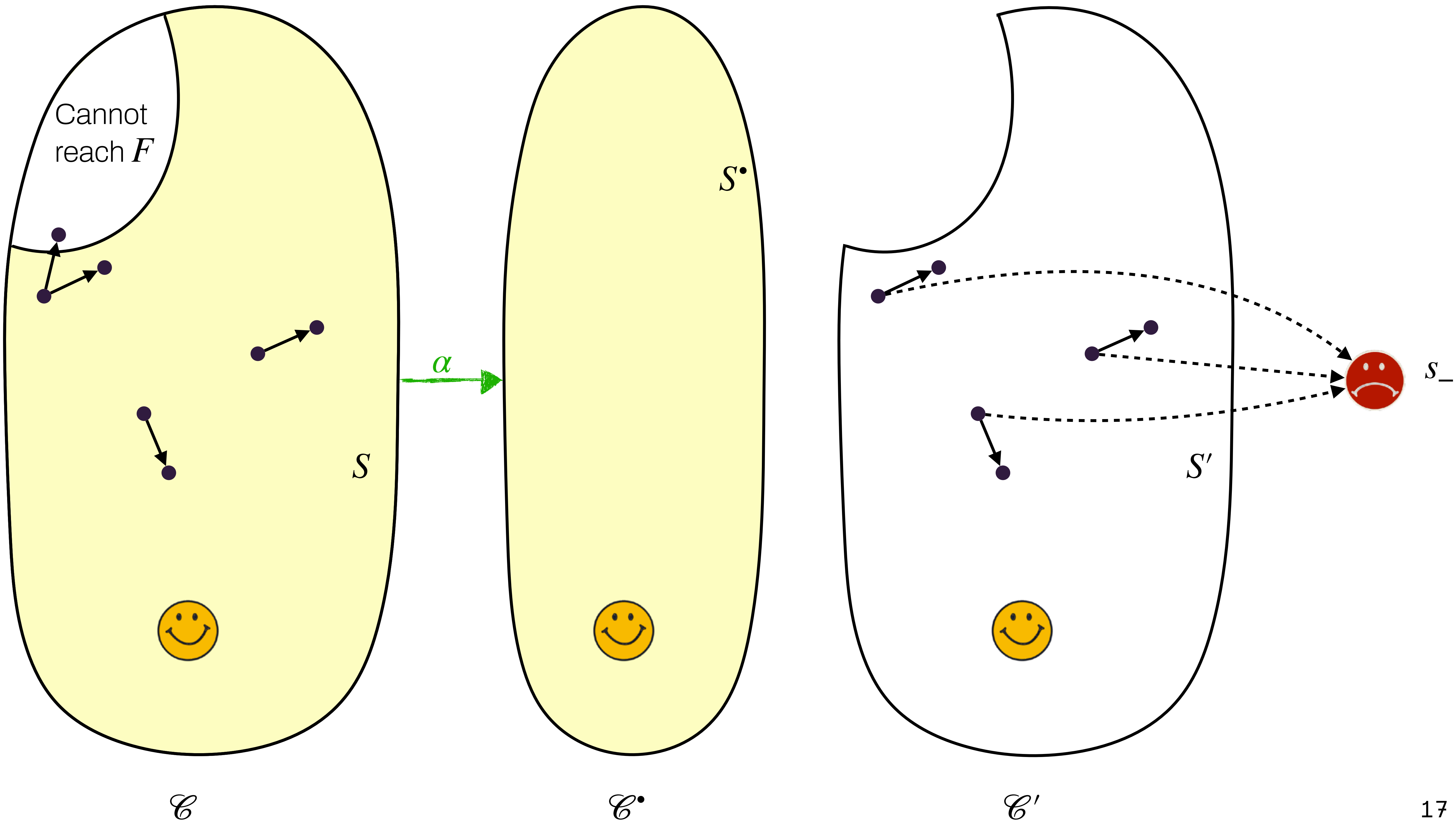

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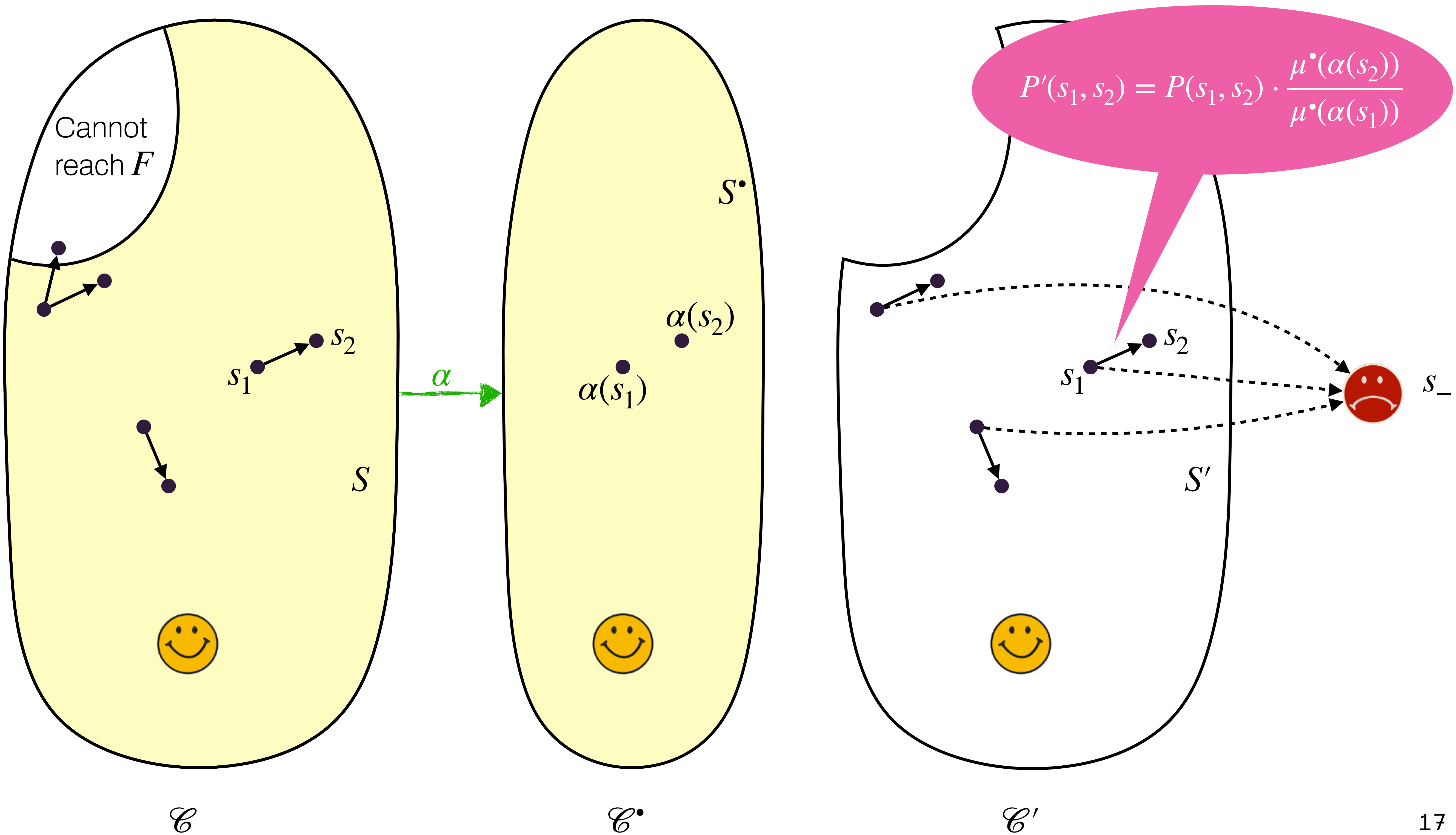

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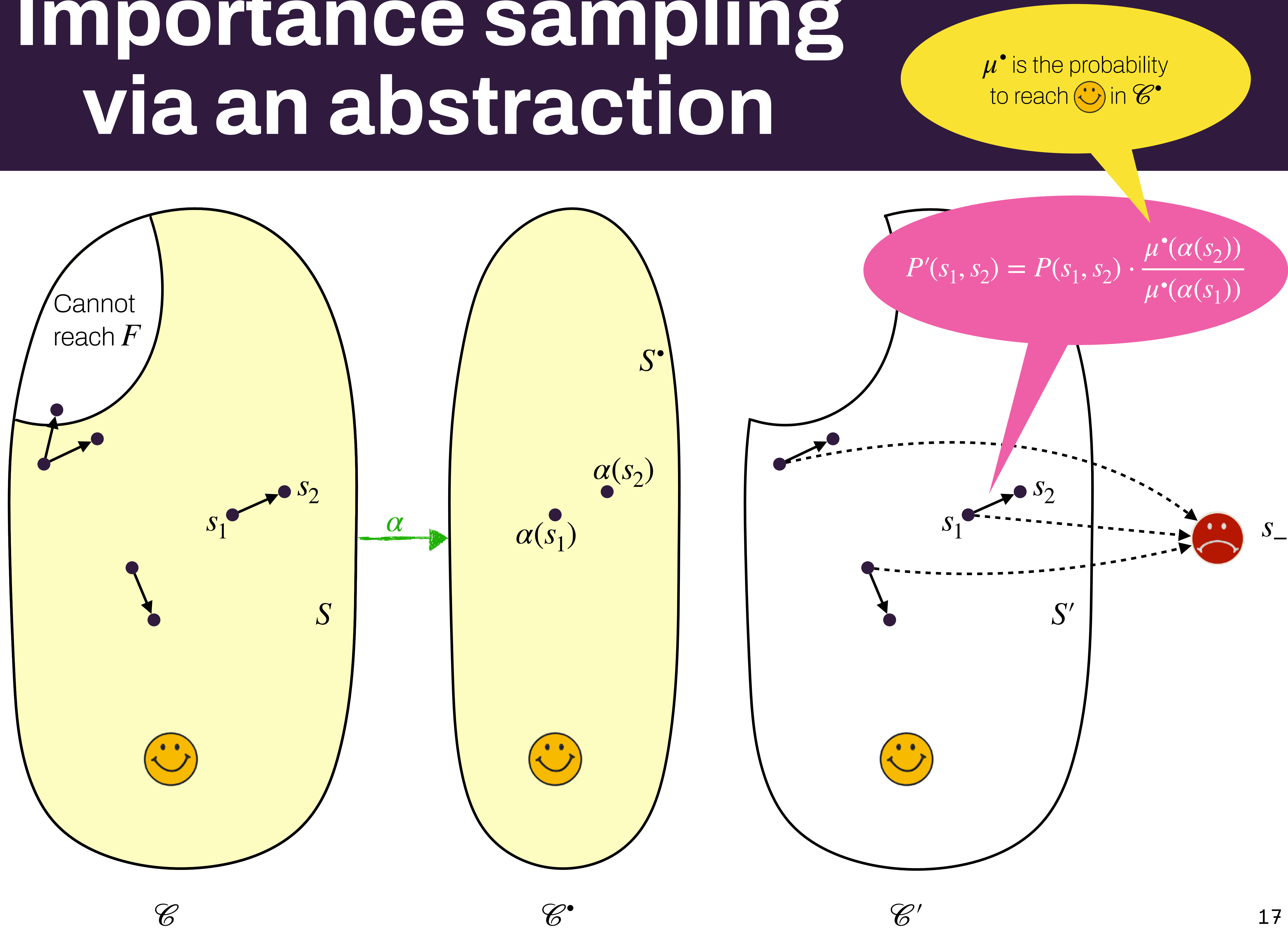

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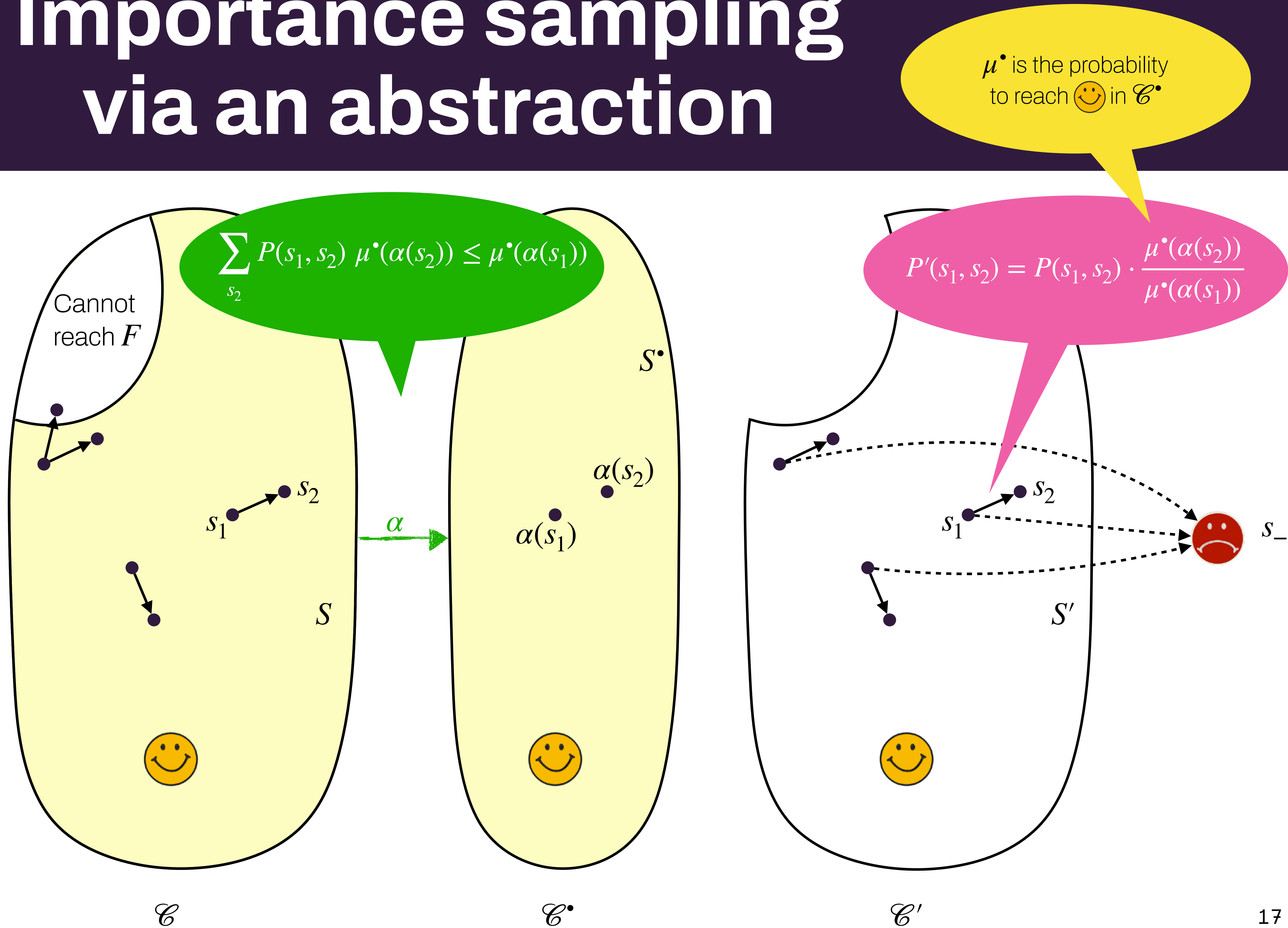
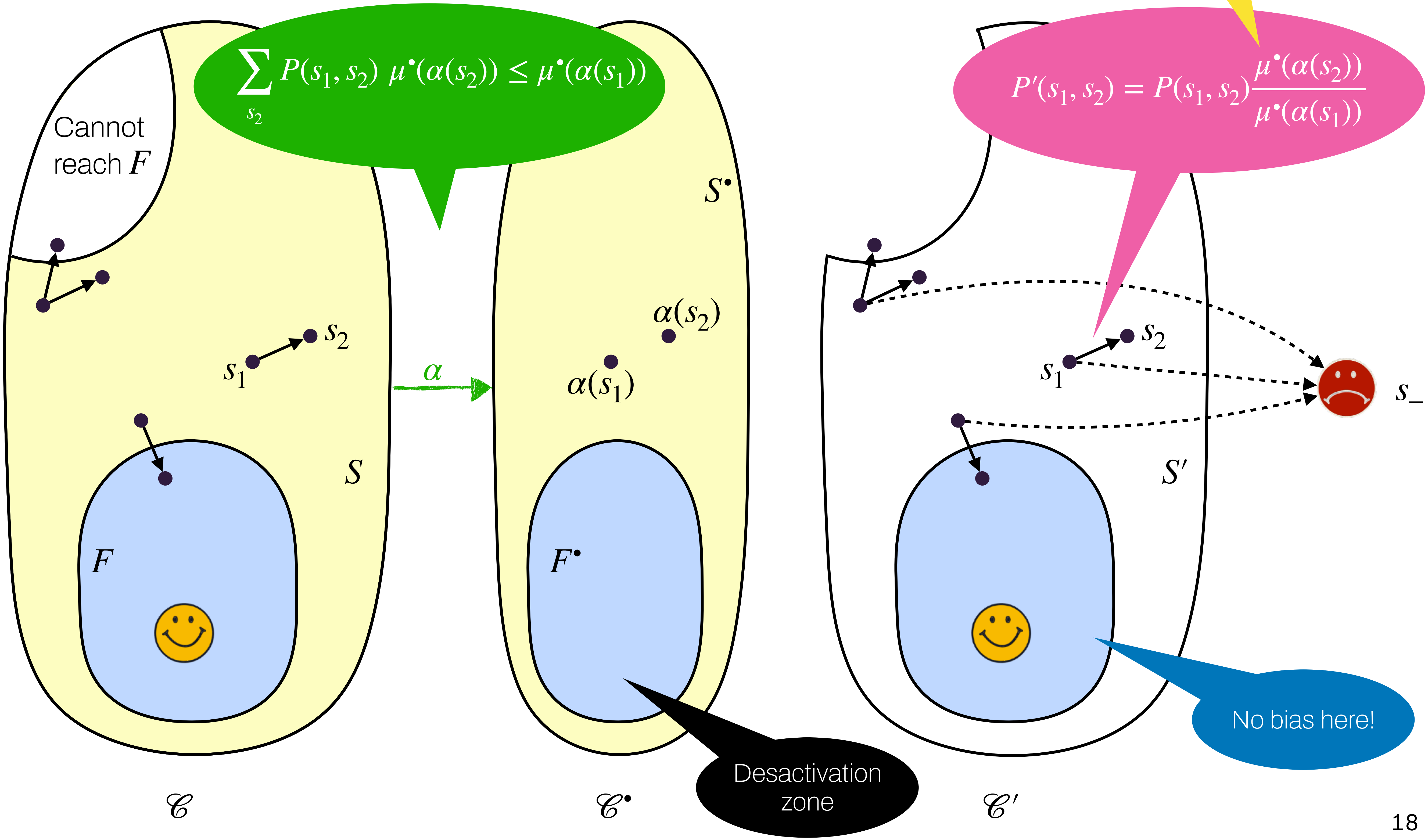
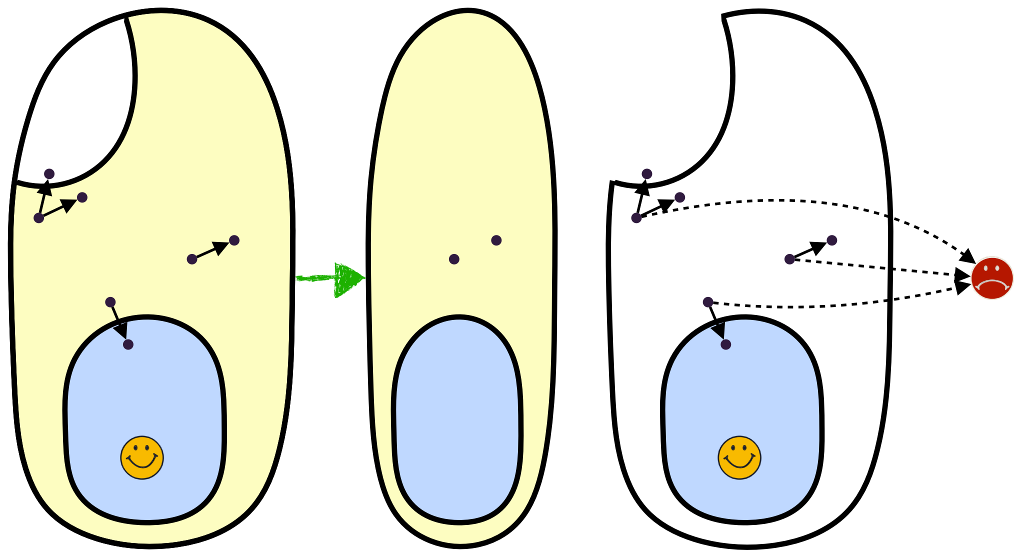


Importance sampling via an abstraction

$\mu^\bullet$ is the probability to reach $F^\bullet$ in $\mathscr{C}^\bullet$



Further properties of the biased Markov chain obtained via an abstraction

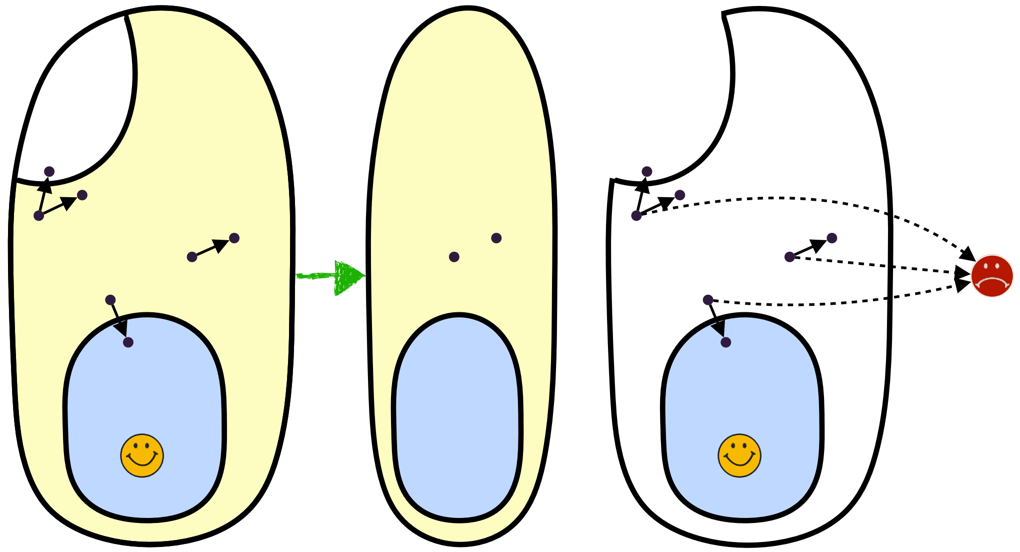


Further properties of the biased Markov chain obtained via an abstraction

Property of the bias

$$\gamma(\rho) = \begin{cases} \mu^\bullet(\alpha(s_0)) & \text{if } \rho \text{ ends in } ☺ \\ 0 & \text{otherwise} \end{cases}$$

It is bivaluated, hence bounded



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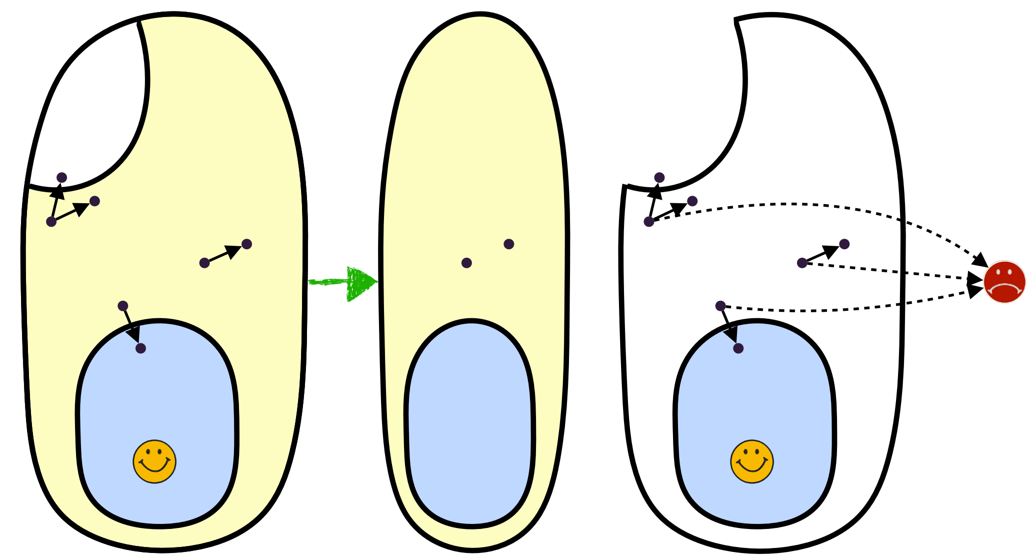

Theorem

If F is finite and for every $0 \le x < 1$, $\{s \in S \mid \mu^\bullet(\alpha(s)) \ge x\}$ is finite, then $\mathscr{C}'$ is decisive w.r.t. ☺.

Proof using attractors, martingale theory

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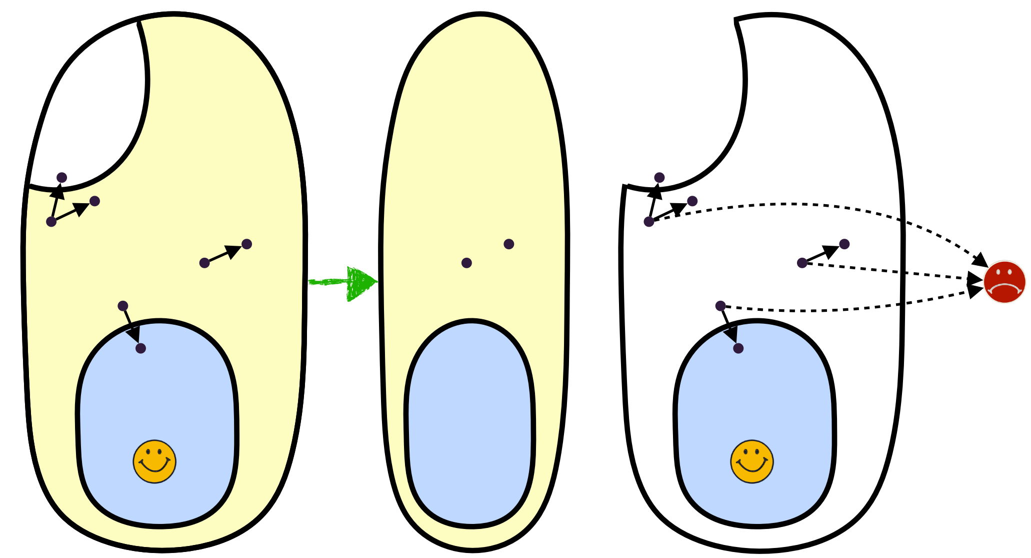

Theorem

If F is finite and for every $0 \leq x < 1$, $\{s \in S \mid \mu^\bullet(\alpha(s)) \geq x\}$ is finite, then $\mathscr{C}'$ is decisive w.r.t. ☺.

Proof using attractors, martingale theory

- The analysis can be performed on $\mathscr{C}'$!

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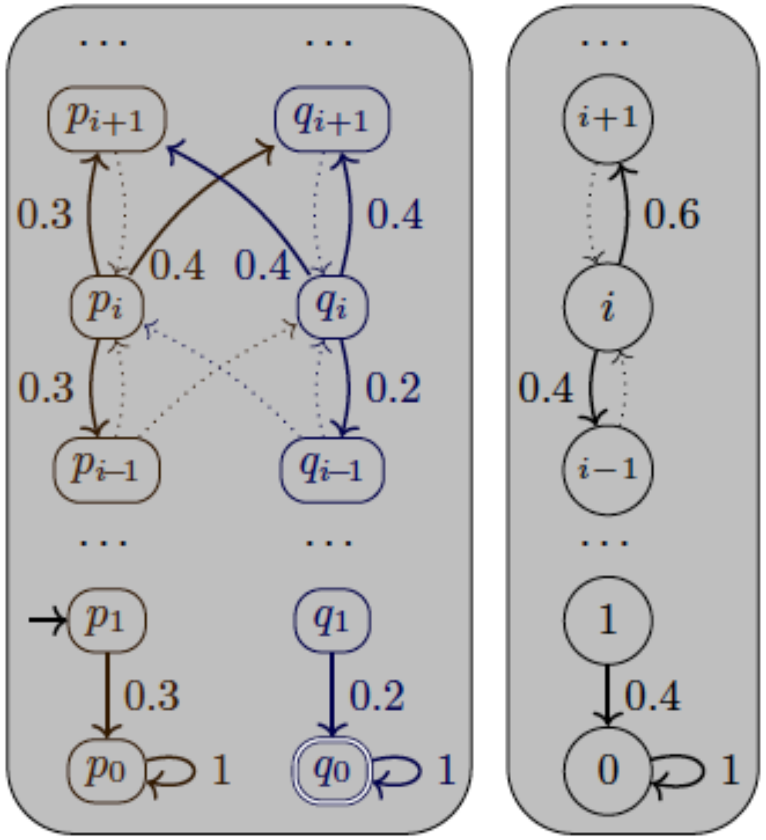

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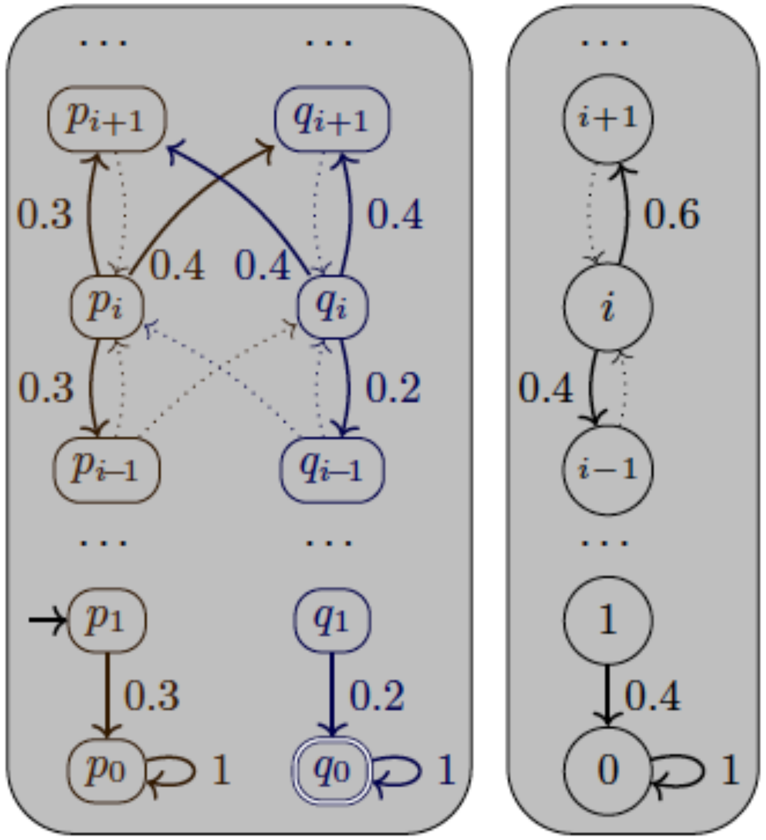

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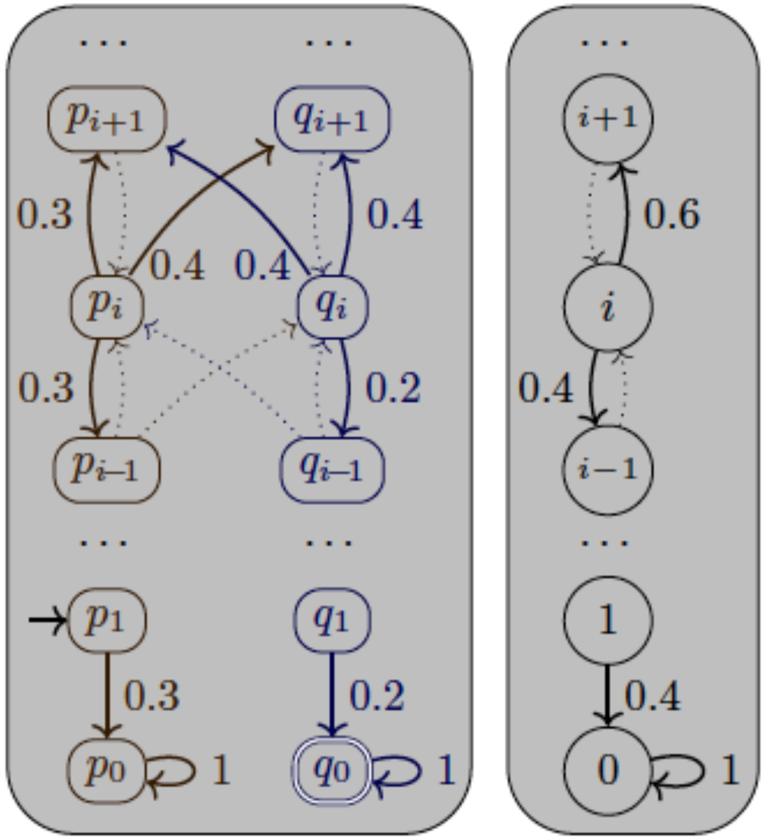

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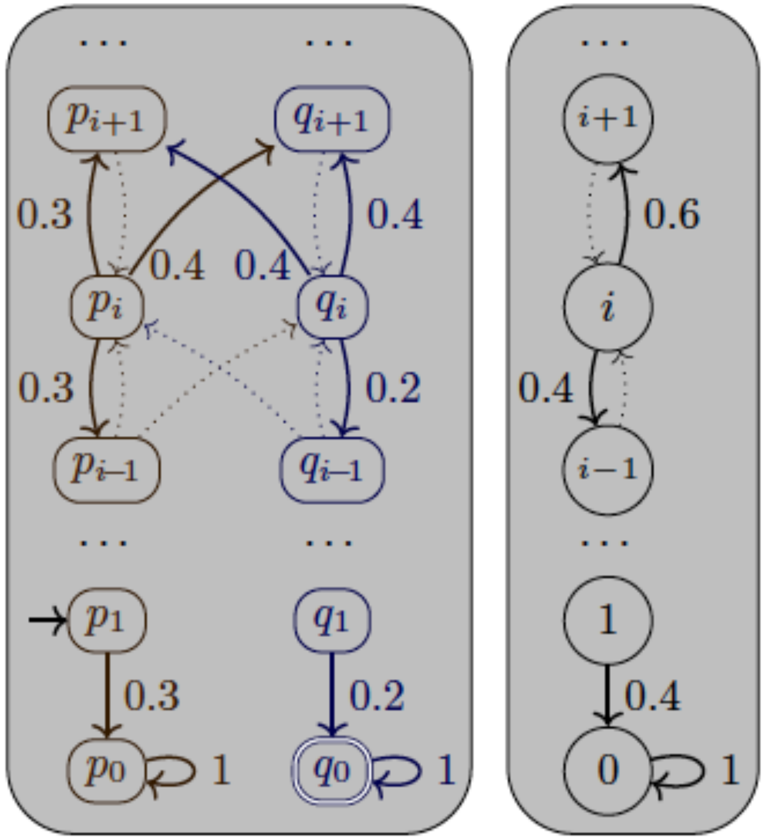

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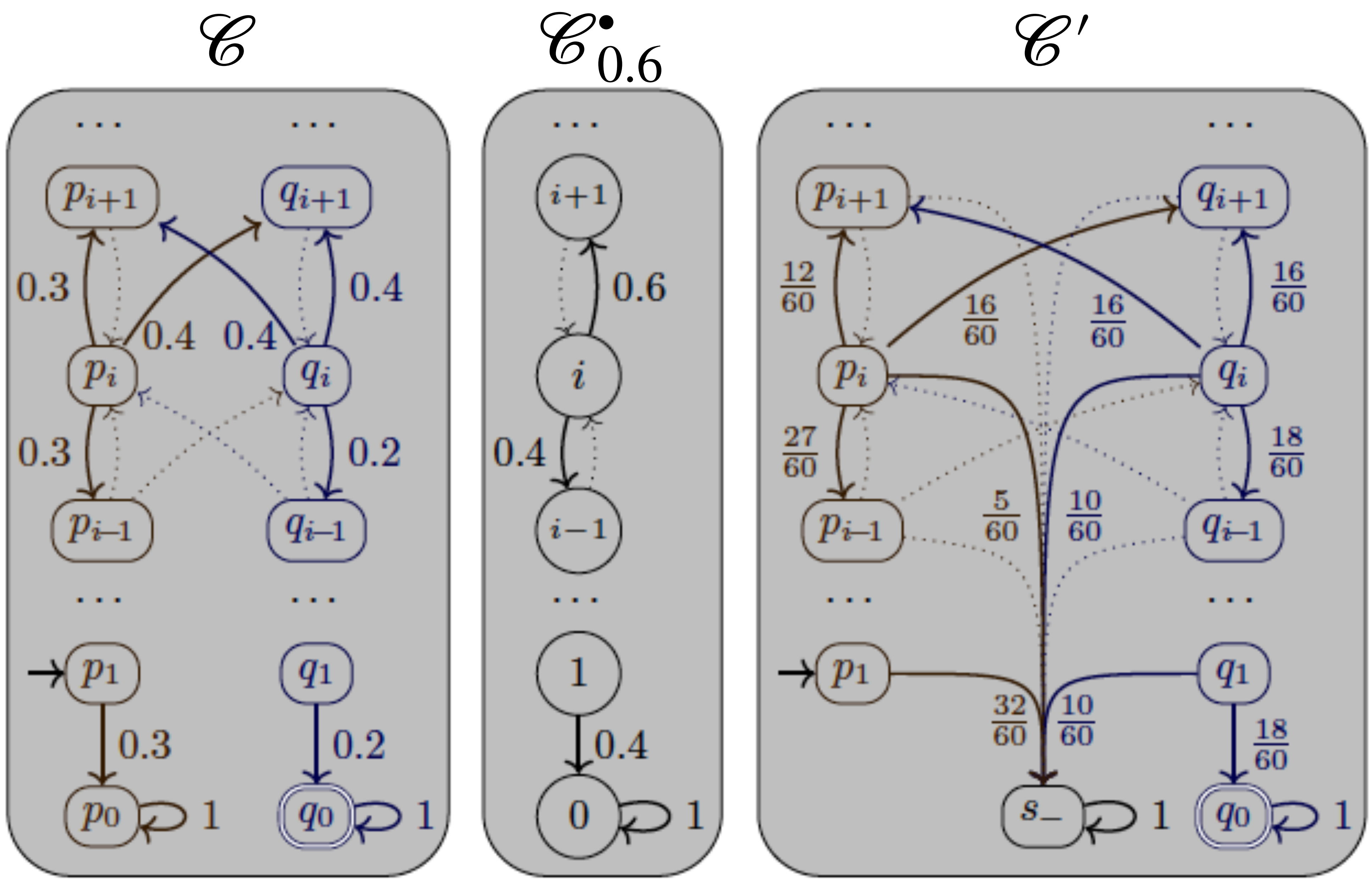
p OK and $\frac{1}{2} < p' < p \Rightarrow p'$ OK

Example



$\mathscr{C}$ is not decisive

$\mathscr{C}'$ is decisive
+ finite sampling time

Implementation and experiments

https://cosmos.lacl.fr/

[BBDHP15] P. Ballarini, B. Barbot, M. Duflot, S. Haddad, N. Pekergin. Hasl: A new approach for performance evaluation and model checking from concepts to experimentation (Performance Evaluation)

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Note: in all experiments, the confidence is set to $99\ \%$

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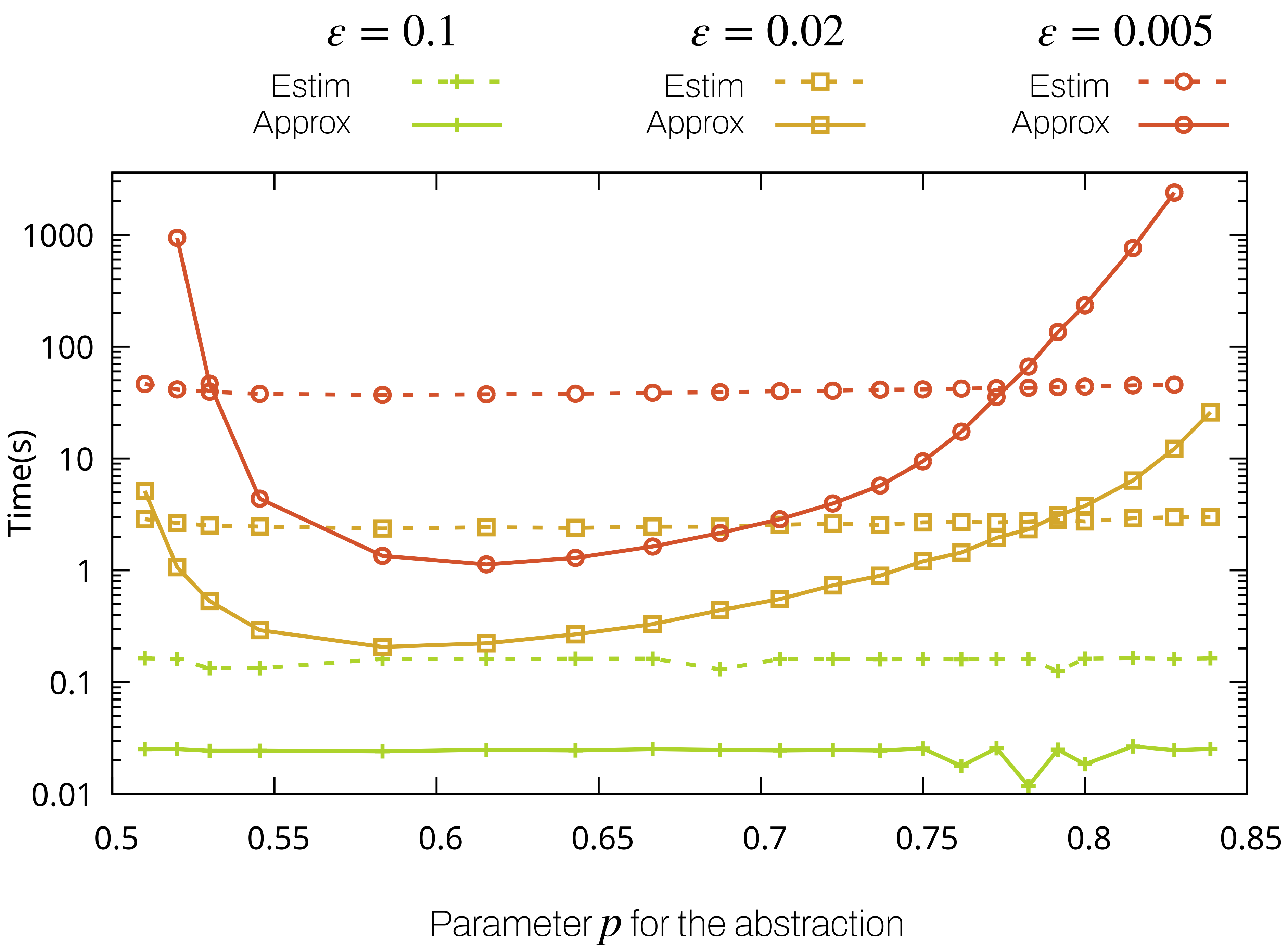
Examples of results



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Any theoretical justification for that?