Laboratoire
Méthodes
Formelles

Beyond Decisiveness: When Statistical Verification Meets Numerical Verification

Patricia Bouyer

Joint work with Benoît Barbot (LACL) and Serge Haddad (LMF)
supported by ANR projects MAVeriQ and BisoUS
(not submitted yet, hopefully soon on ArXiV)

Purpose of this work

Design algorithms to estimate probabilities in some **infinite-state** Markov chains, **with guarantees**

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Design algorithms to estimate probabilities in some **infinite-state** Markov chains, **with guarantees**

Our contributions

- Review two existing approaches (approximation algorithm and estimation algorithm) and specify the required hypothesis for correctness
- Propose an approach based on **importance sampling** and **abstraction** to partly relax the hypothesis
- Analyze empirically the approaches

Discrete-time Markov chains

Discrete-time Markov chain (DTMC)

$\mathscr{C} = (S, s_0, \delta)$ with S at most denumerable, $s_0 \in S$ and $\delta : S \to \mathrm{Dist}(S)$

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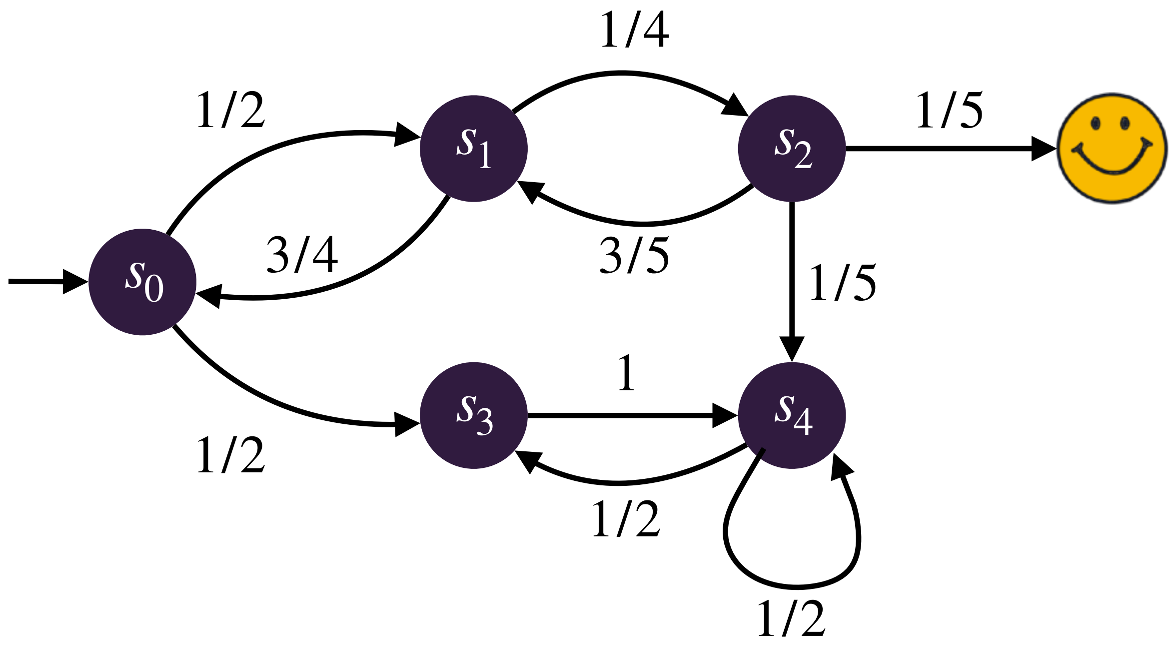

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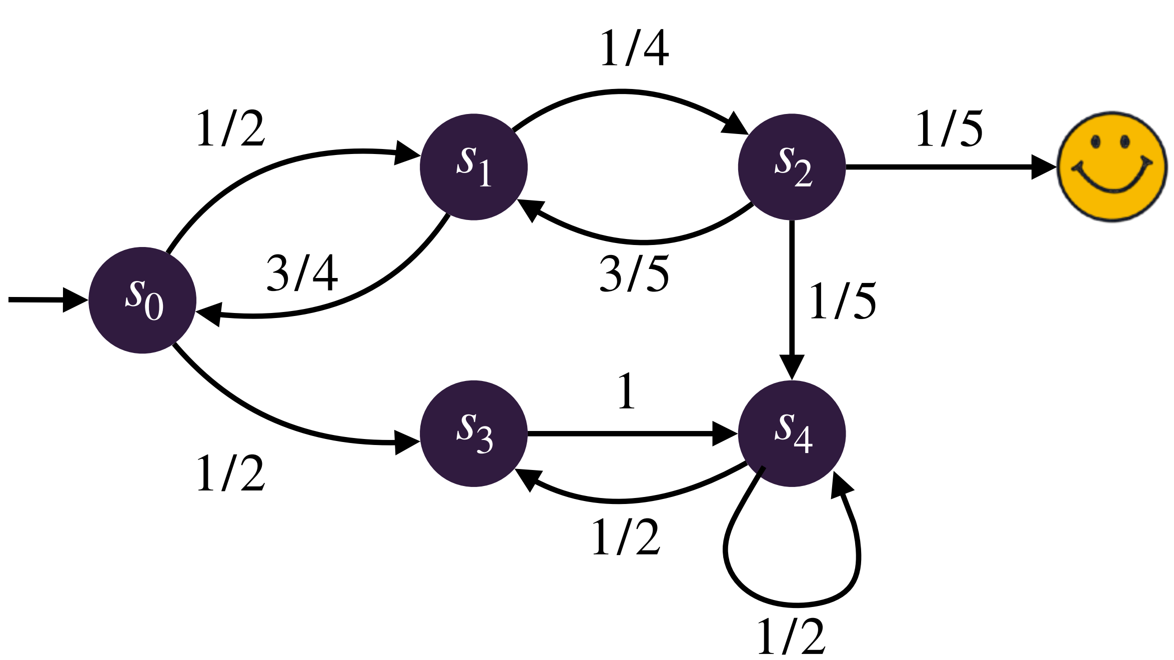

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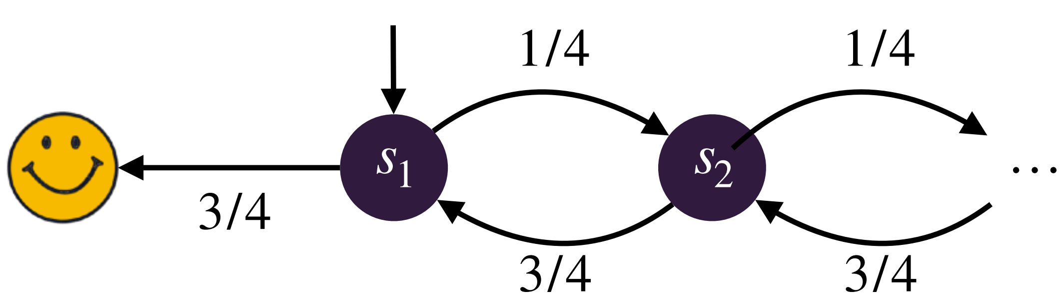

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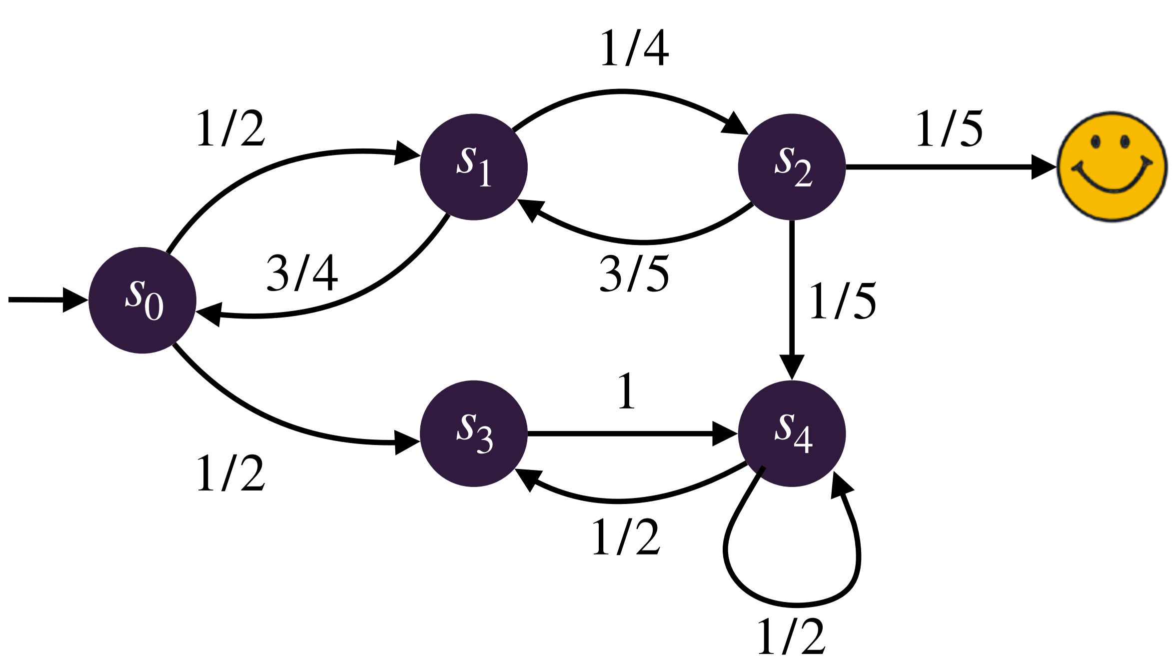

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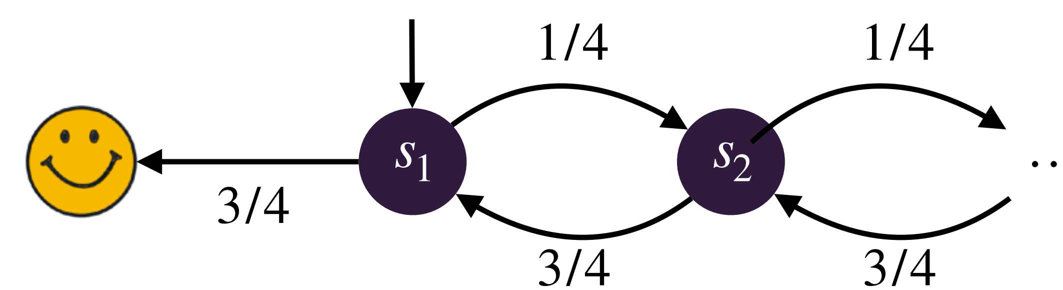

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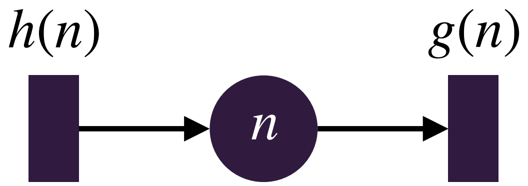
High-level models for (infinite) Markov chains

- Queues



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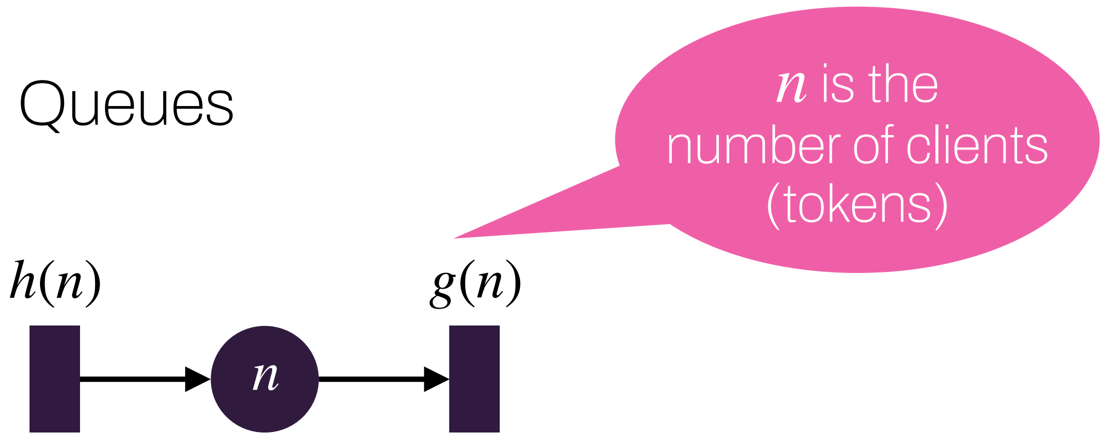

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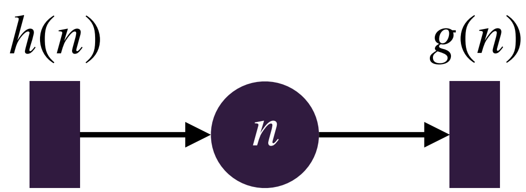

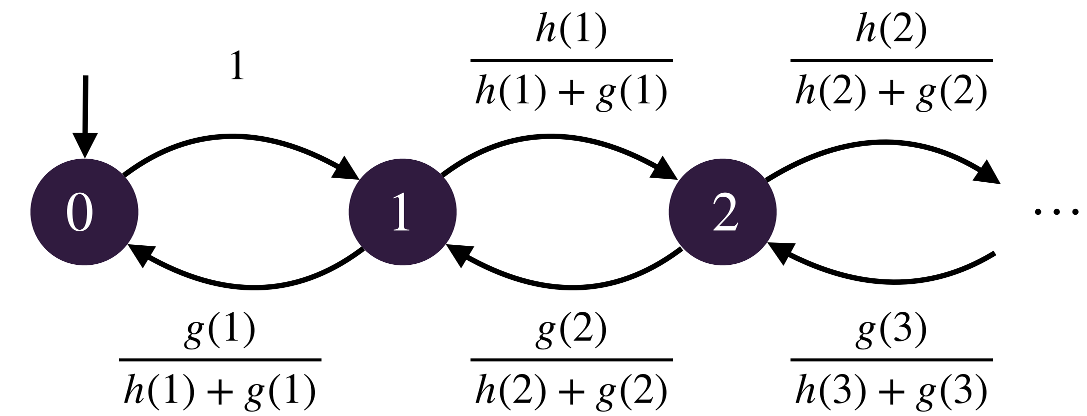

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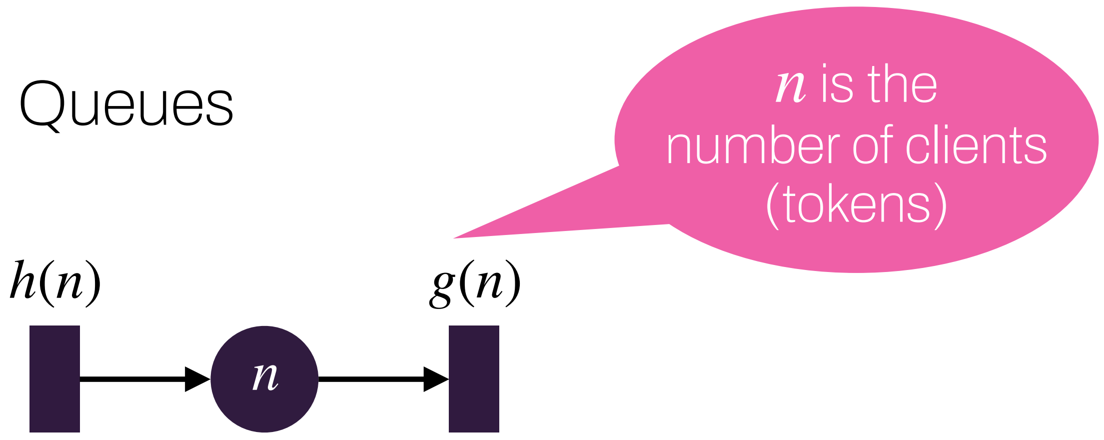

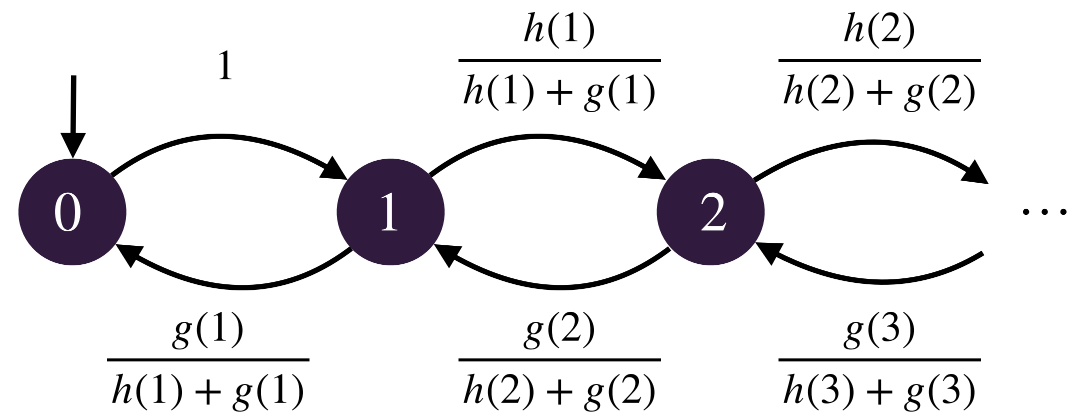

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$$A \xrightarrow{1} C \quad A \xrightarrow{n} BB \quad B \xrightarrow{5} \varepsilon$$
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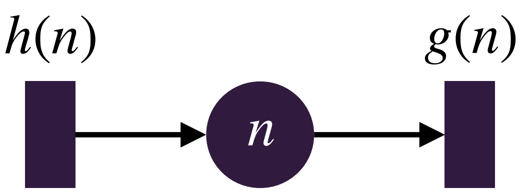

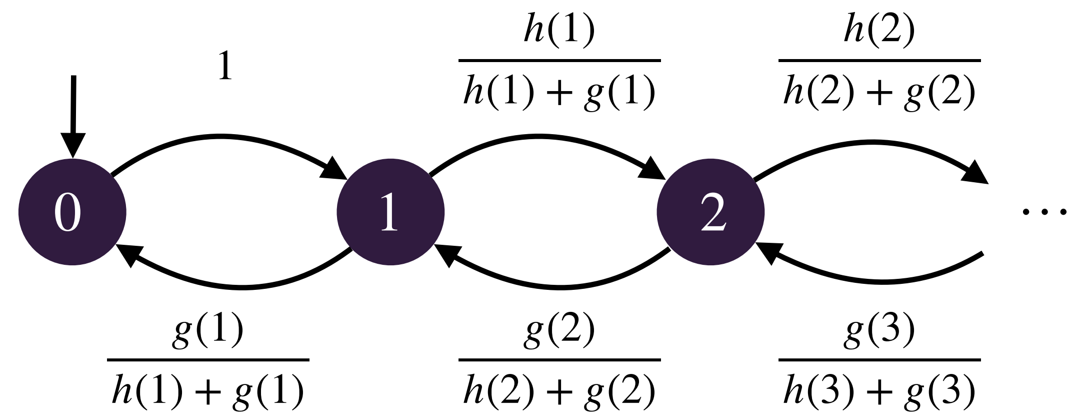

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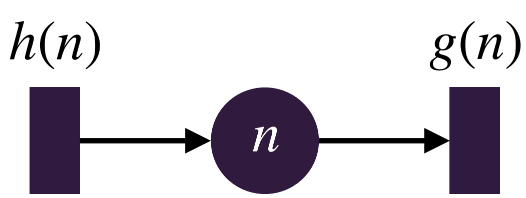

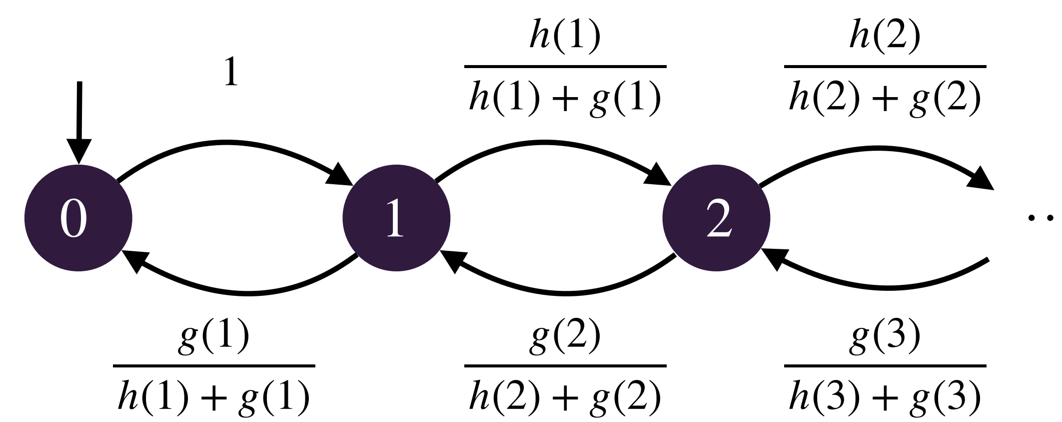

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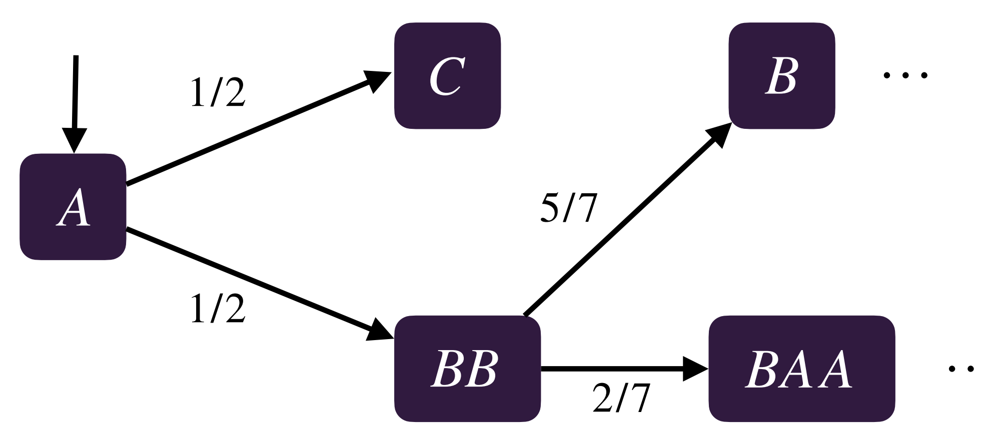

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Very useful even beyond reachability properties (decomposition in BSCCs)

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- For state s, let x_s be such that:

$x_s = $
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- Specific approaches for **decisive** Markov chains

[EKM06] J. Esparza, A. Kucera, R. Mayr. Model Checking Probabilistic Pushdown Automata (LMCS)

Decisiveness

 $= \{s \in S \mid s \not\models \exists \mathbf{F}$  $\}$

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A DTMC $\mathscr{C}$ is decisive from s w.r.t.  if $\mathbb{P}_s(\mathbf{F}$  $\vee\, \mathbf{F}$  $) = 1$

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)

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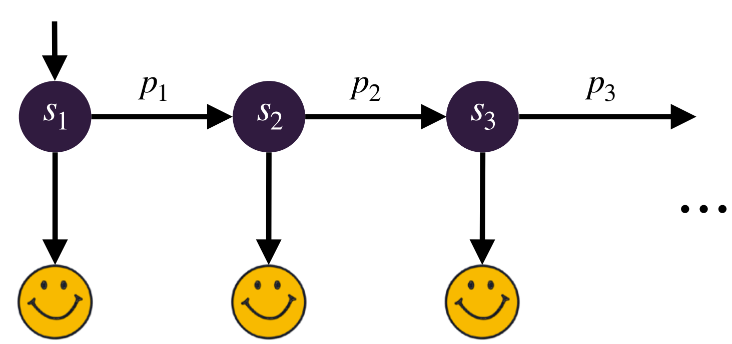

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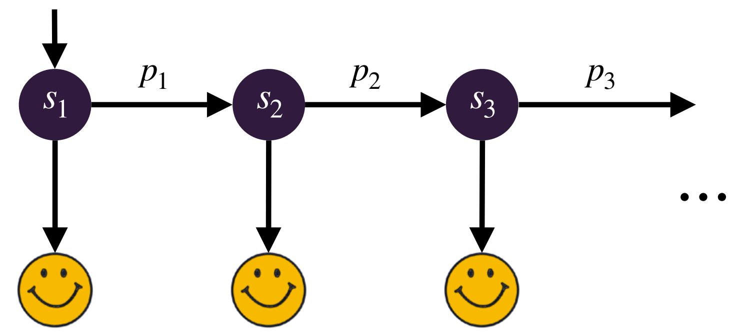

- $\mathbb{P}(\mathbf{G}\neg$ 🙂$) = \prod_{i \geq 1} p_i$
- Decisive iff this product equals 0

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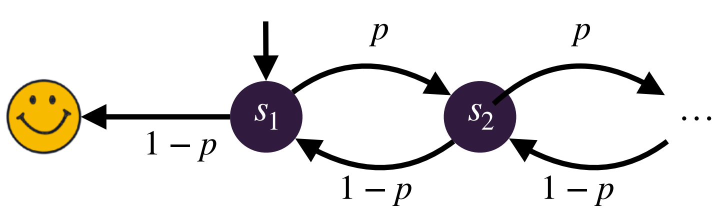

- Recurrent random walk ($p \leq 1/2$): decisive
- Transient random walk ($p > 1/2$): not decisive

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)

Deciding decisiveness?

Classes where decisiveness can be decided

- Probabilistic pushdown automata with constant weights [ABM07]
- Random walks with polynomial weights [FHY23]
- So-called probabilistic homogeneous one-counter machines with polynomial weights (this extends the model of quasi-birth death processes) [FHY23]

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)
[FHY23] A. Finkel, S. Haddad, L. Yé. About decisiveness of dynamic probabilistic models (CONCUR'23)

Approximation scheme

- Aim: compute probability of $\mathbf{F}$

[IN97] P. Iyer, M. Narasimha. Probabilistic lossy channel systems (TAPSOFT'97)

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until $p_n^{\text{yes}} + p_n^{\text{no}} \geq 1 - \varepsilon$

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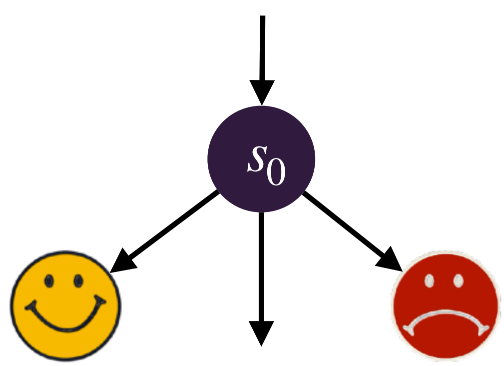

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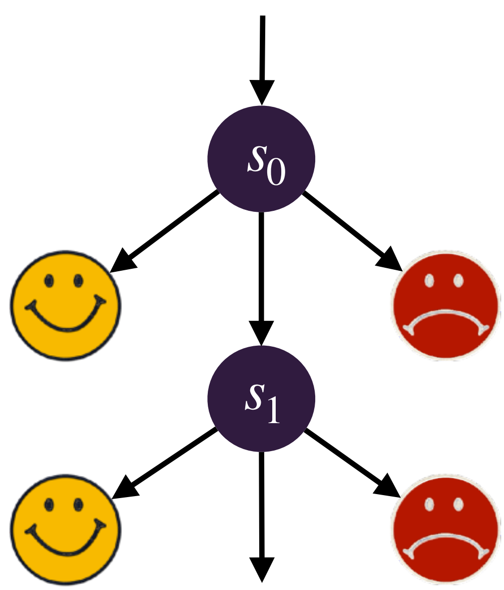

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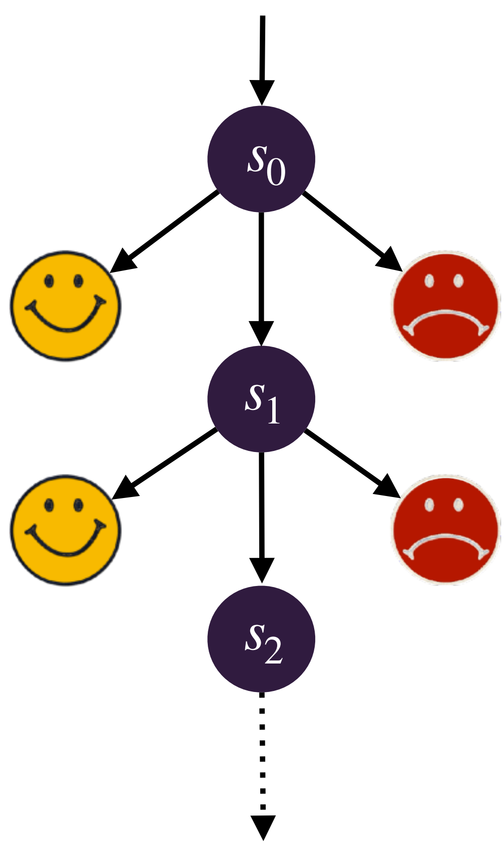

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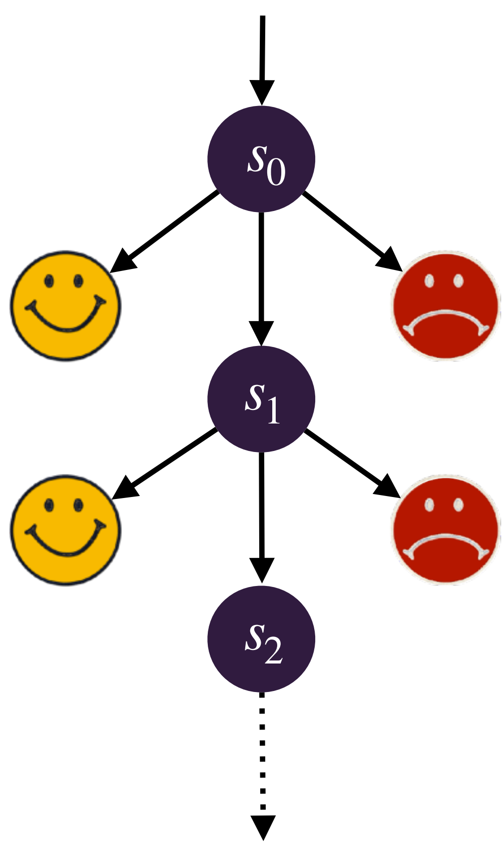

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$$p_2^{\text{yes}} \leq \mathbb{P}(\mathbf{F} ☺) \leq 1 - p_2^{\text{no}}$$

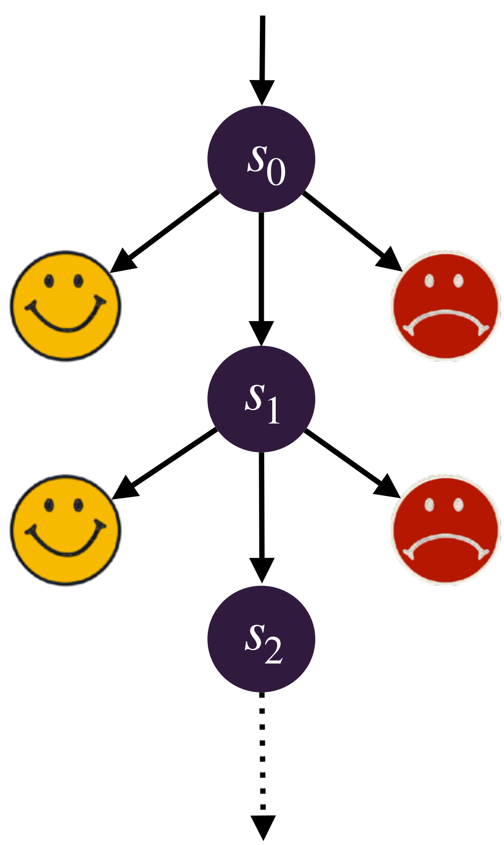
$$\text{IV} \qquad \vdots \qquad \text{VI}$$

Does it converge?

[IN97] P. Iyer, M. Narasimha. Probabilistic lossy channel systems (TAPSOFT'97)

Approximation scheme

- Aim: compute probability of $\mathbf{F}\,\smiley$
- $\frownie = \{s \in S \mid s \not\models \exists\mathbf{F}\,\smiley\}$



Approximation scheme

Given $\varepsilon > 0$, for every n, compute:

$$\begin{cases} p_n^{\text{yes}} = \mathbb{P}(\mathbf{F}_{\leq n}\,\smiley) \\ p_n^{\text{no}} = \mathbb{P}(\neg\,\smiley\,\mathbf{U}_{\leq n}\,\frownie) \end{cases}$$

until $p_n^{\text{yes}} + p_n^{\text{no}} \geq 1 - \varepsilon$

$$p_1^{\text{yes}} \leq \mathbb{P}(\mathbf{F}\,\smiley) \leq 1 - p_1^{\text{no}}$$

$$\leq \qquad\qquad \geq$$

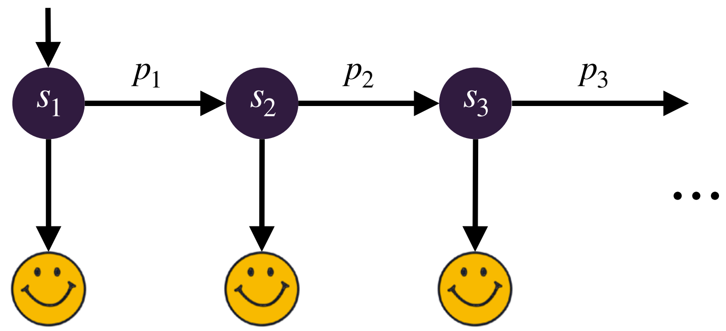
$$p_2^{\text{yes}} \leq \mathbb{P}(\mathbf{F}\,\smiley) \leq 1 - p_2^{\text{no}}$$

$$\leq \qquad \vdots \qquad \geq$$

$$\lim_{n\to\infty} p_n^{\text{Yes}} = \mathbb{P}(\mathbf{F}\,\smiley)$$

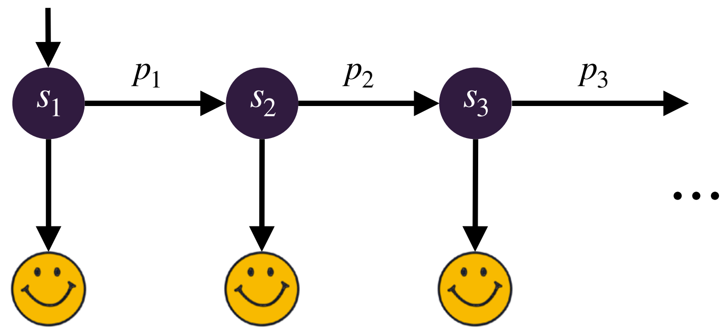
[IN97] P. Iyer, M. Narasimha. Probabilistic lossy channel systems (TAPSOFT'97)

Non-converging example



with $\prod_{i \geq 1} p_i > 0$

Non-converging example



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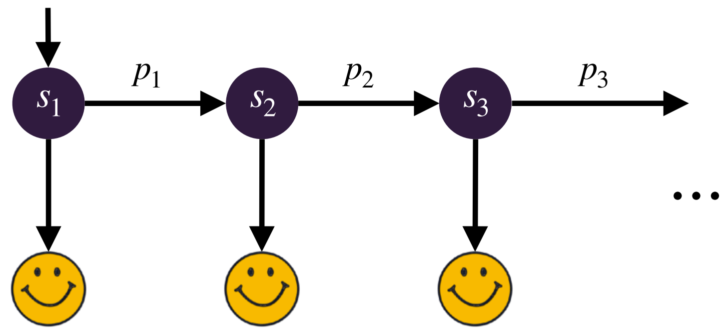

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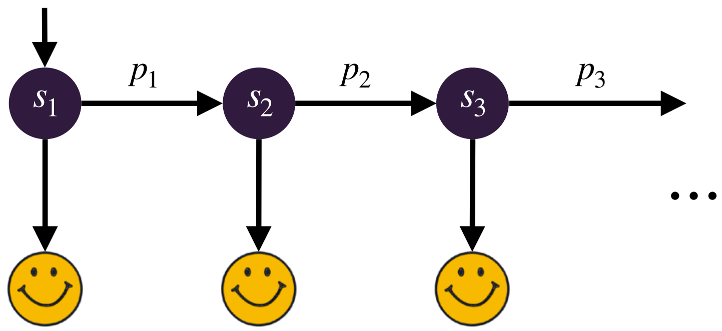

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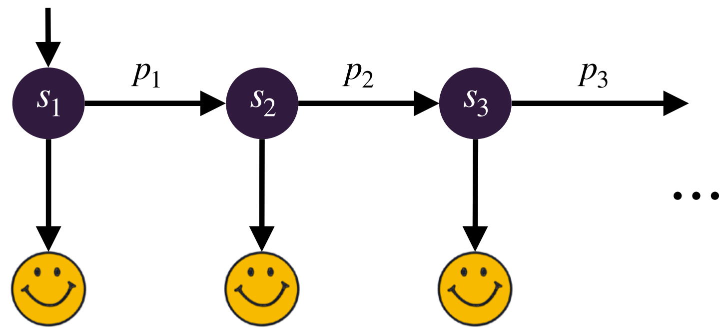

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-  $= \varnothing$
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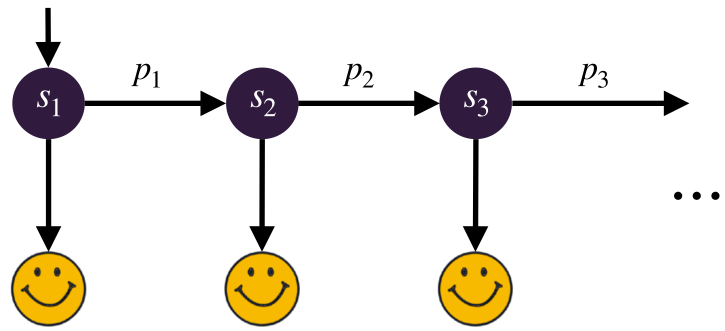

with $\prod_{i \geq 1} p_i > 0$

-  $= \emptyset$
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The approximation scheme does not converge

Termination of the approx. scheme

Approximation scheme

Given $\varepsilon > 0$:

$$\begin{cases} p_n^{\text{yes}} & = & \mathbb{P}(\mathbf{F}_{\leq n} \text{☺}) \\ p_n^{\text{no}} & = & \mathbb{P}(\neg \text{☺} \, \mathbf{U}_{\leq n} \text{☹}) \end{cases}$$

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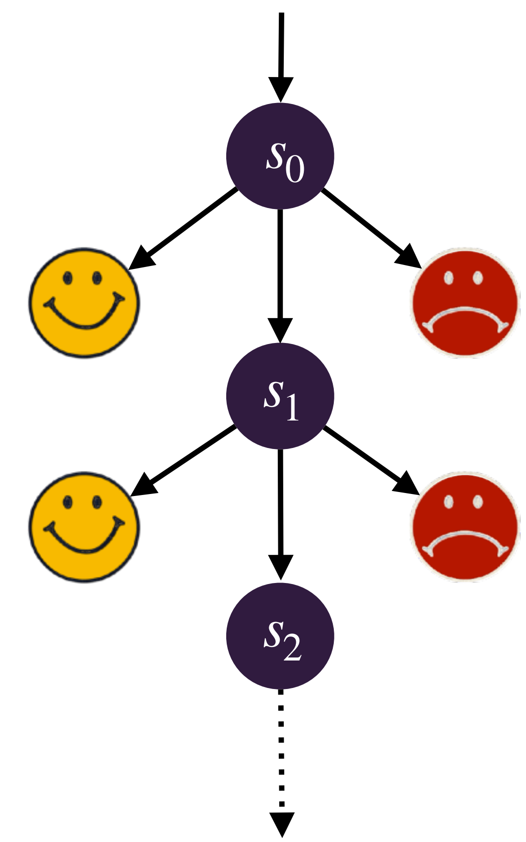

[ABM07] P.A. Abdulla, N. Ben Henda, R. Mayr. Decisive Markov chains (LMCS, 2007)

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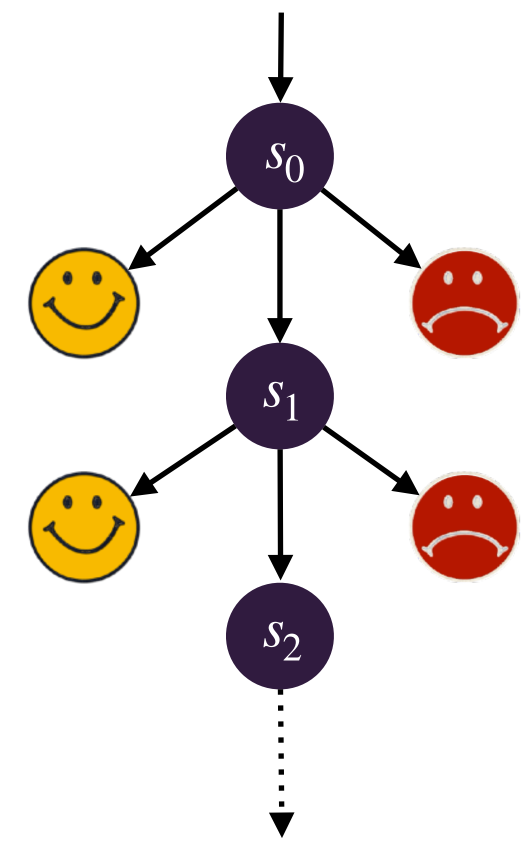

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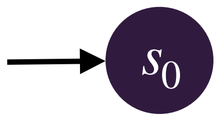
Statistical model-checking

Sample N paths



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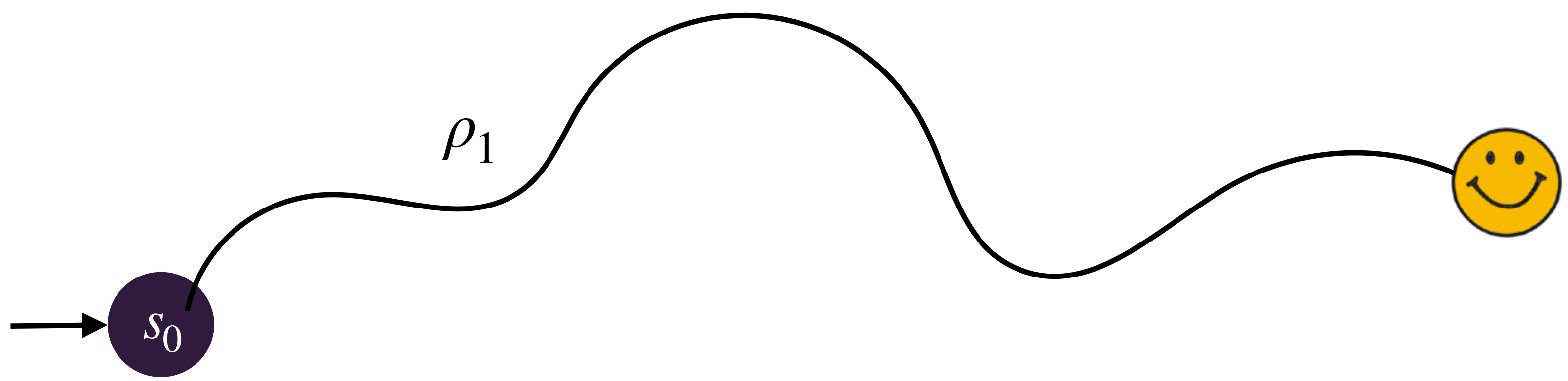

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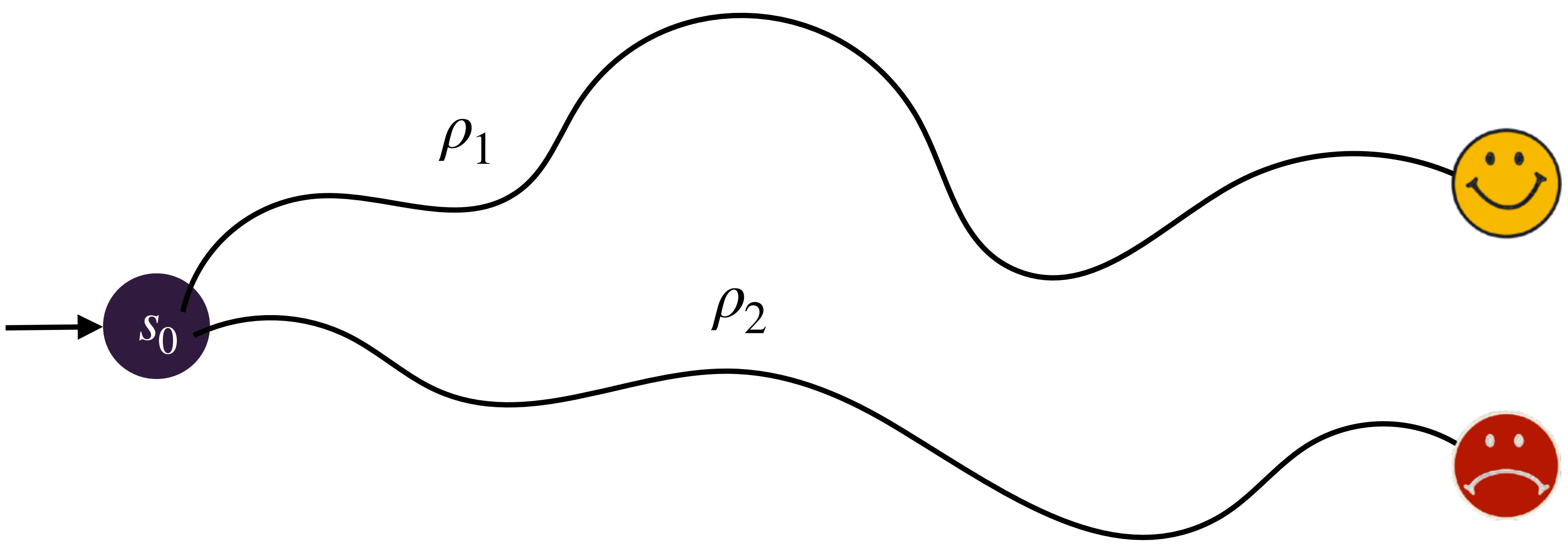

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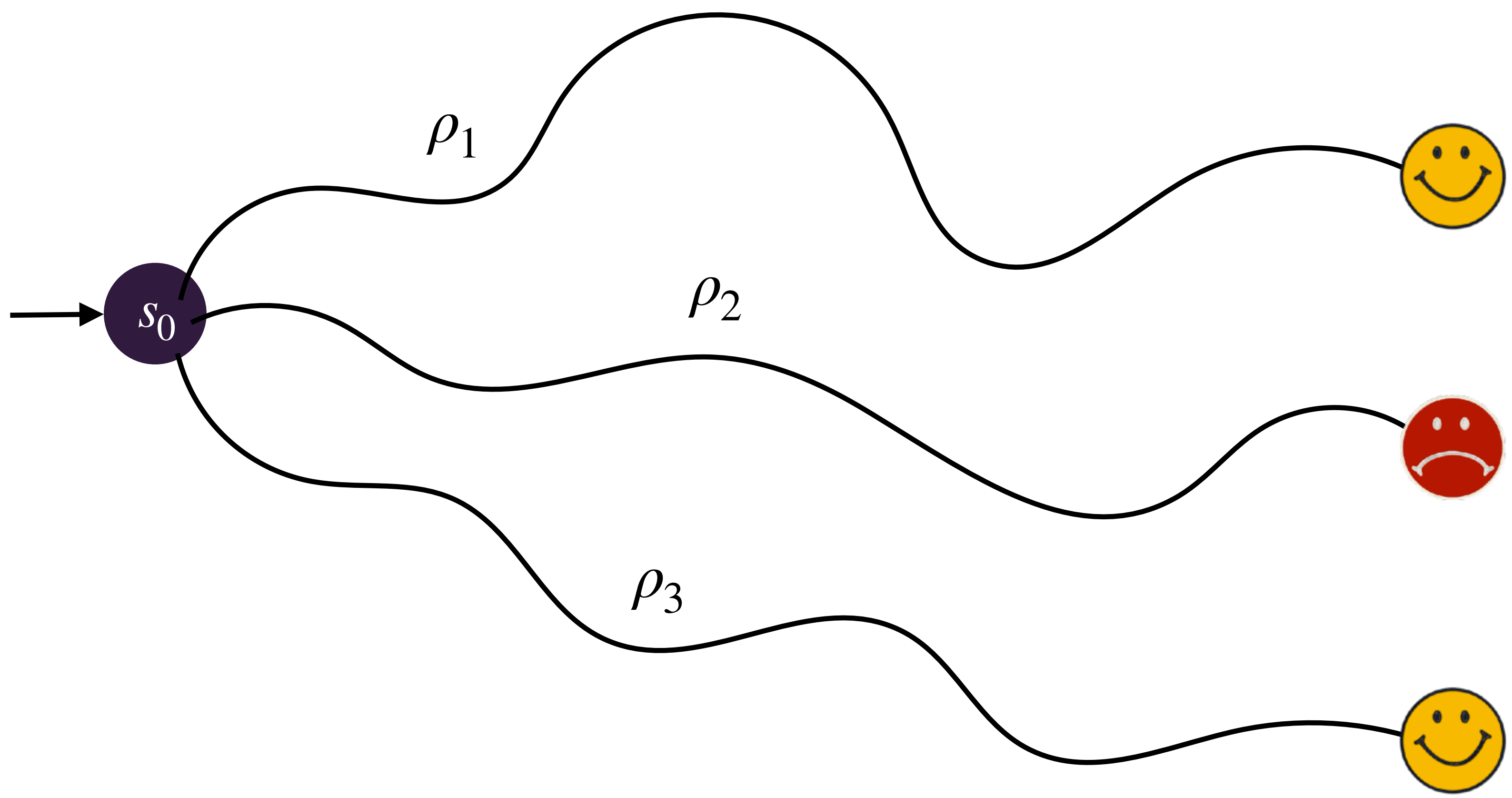

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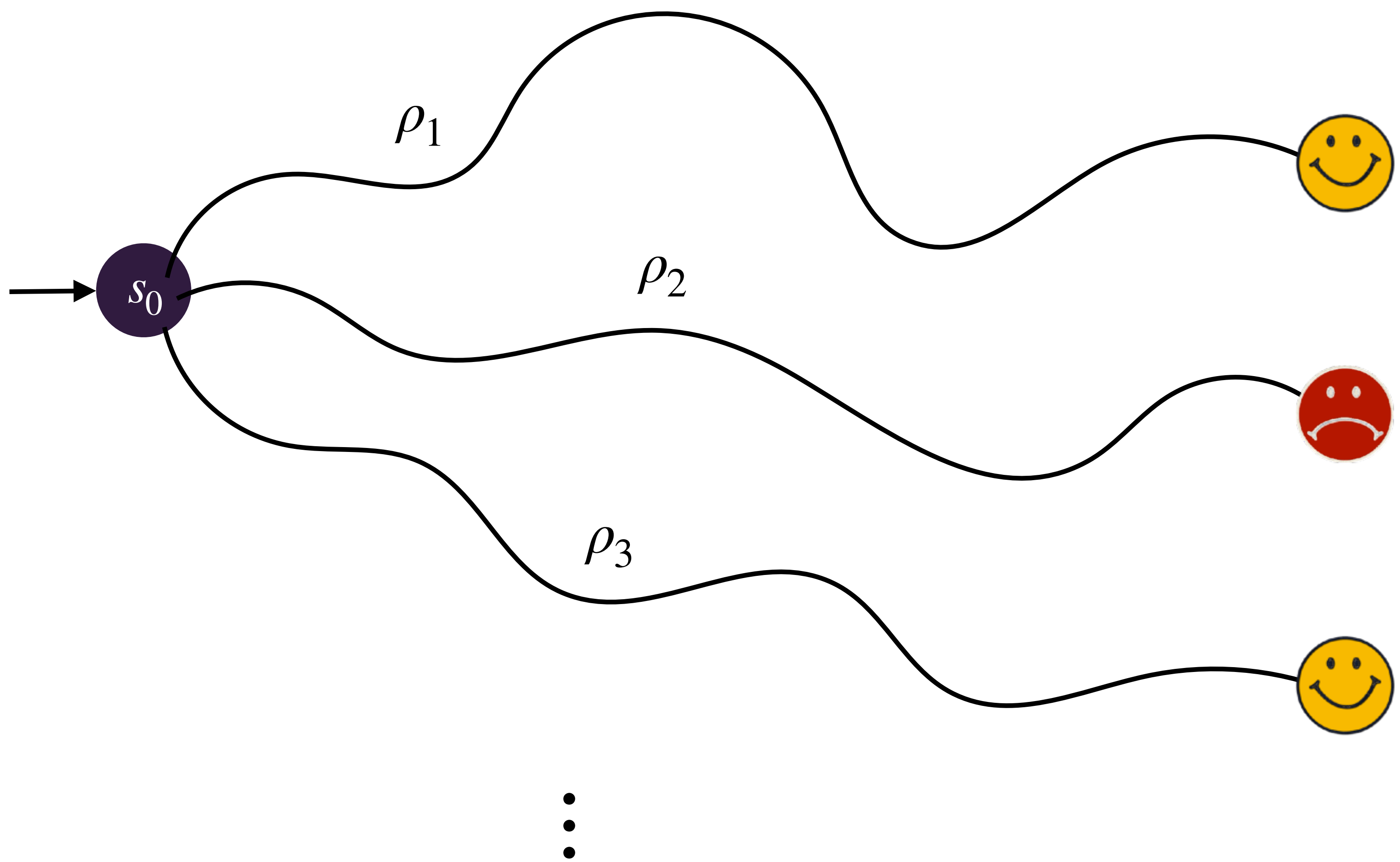

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⋮

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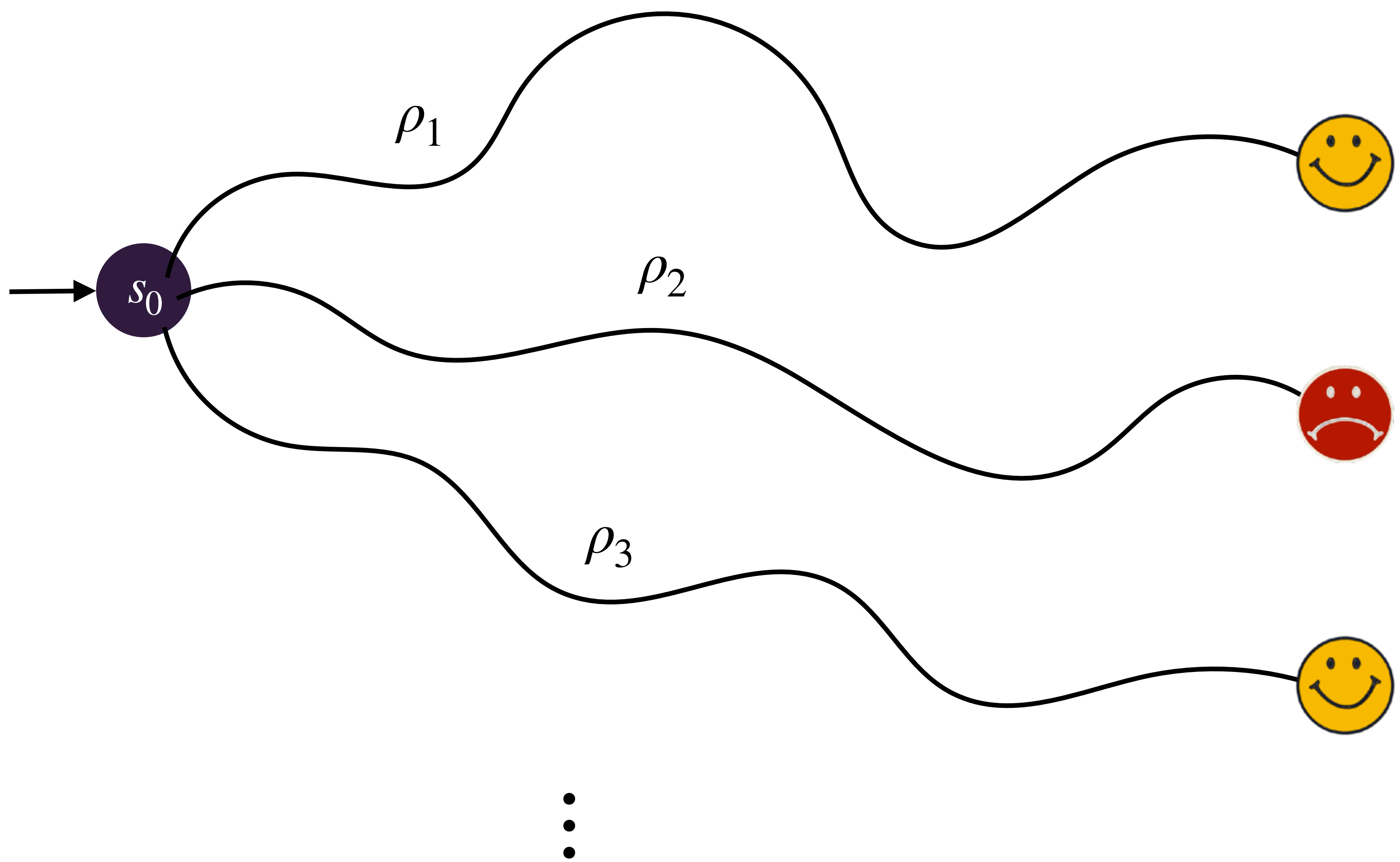

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⋮

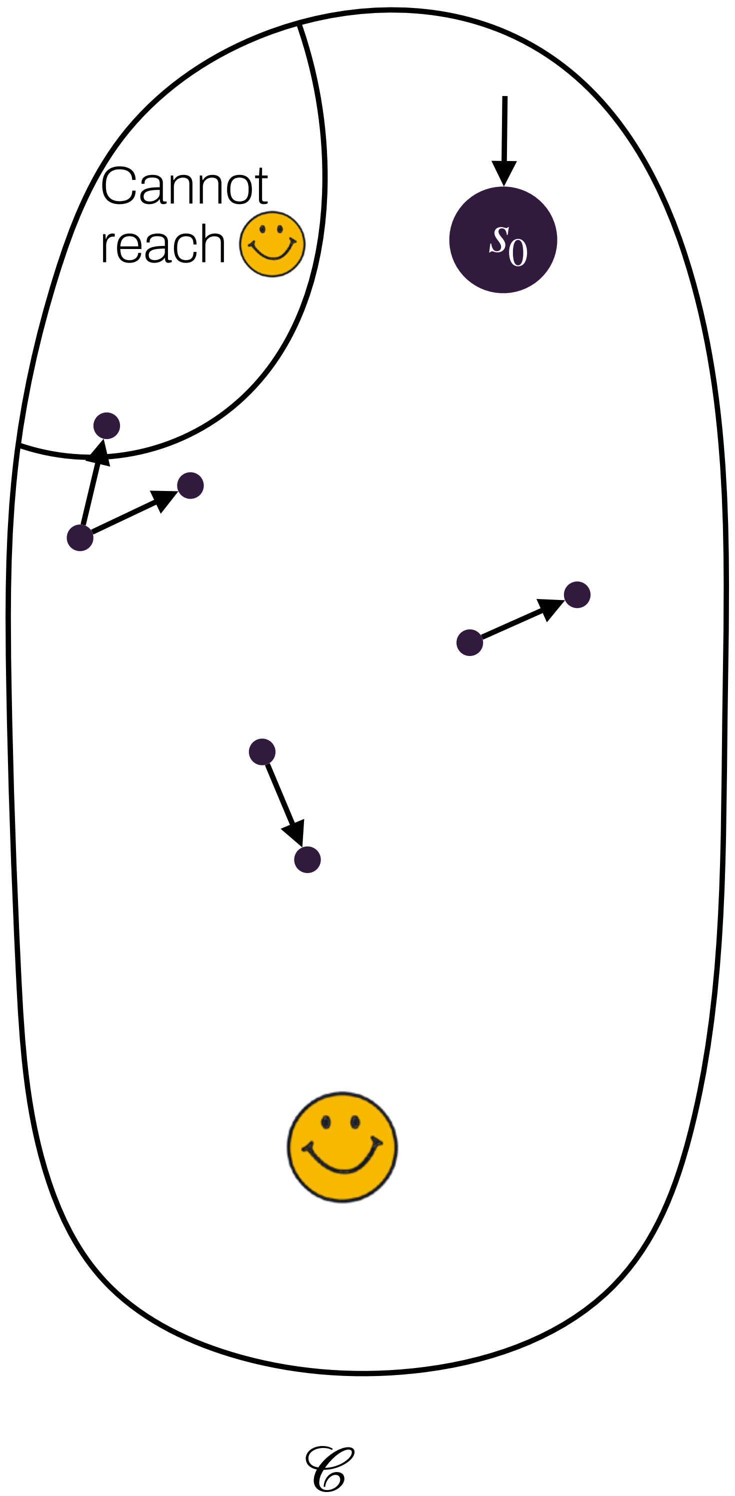
Return $\frac{n_N}{N}$ + some confidence interval

Termination and efficiency

Termination (To our knowledge, never expressed like this)

$\mathscr{C}$ is decisive from s_0 w.r.t. 
iff
a sampled path starting at s_0 almost-surely hits  or 

Decisiveness vs recurrence



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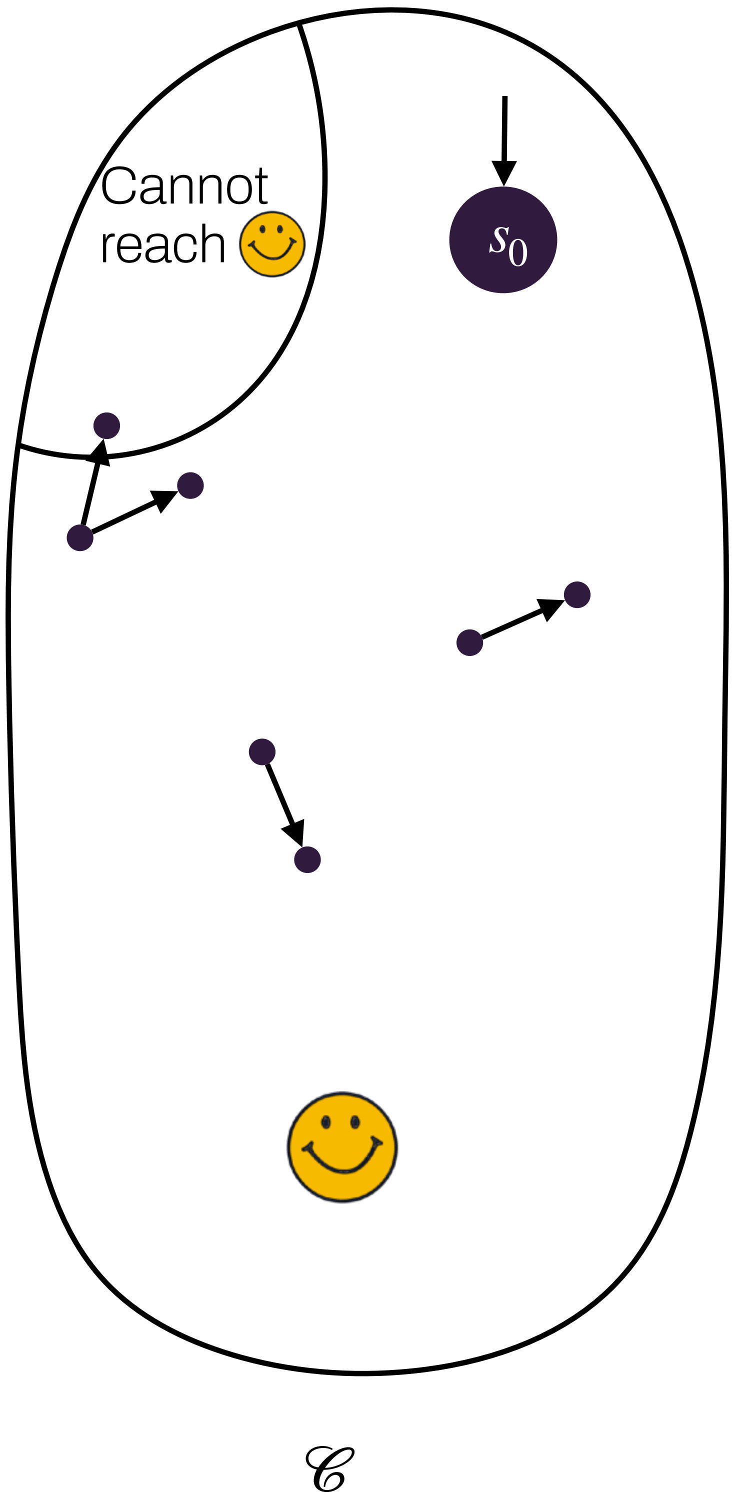

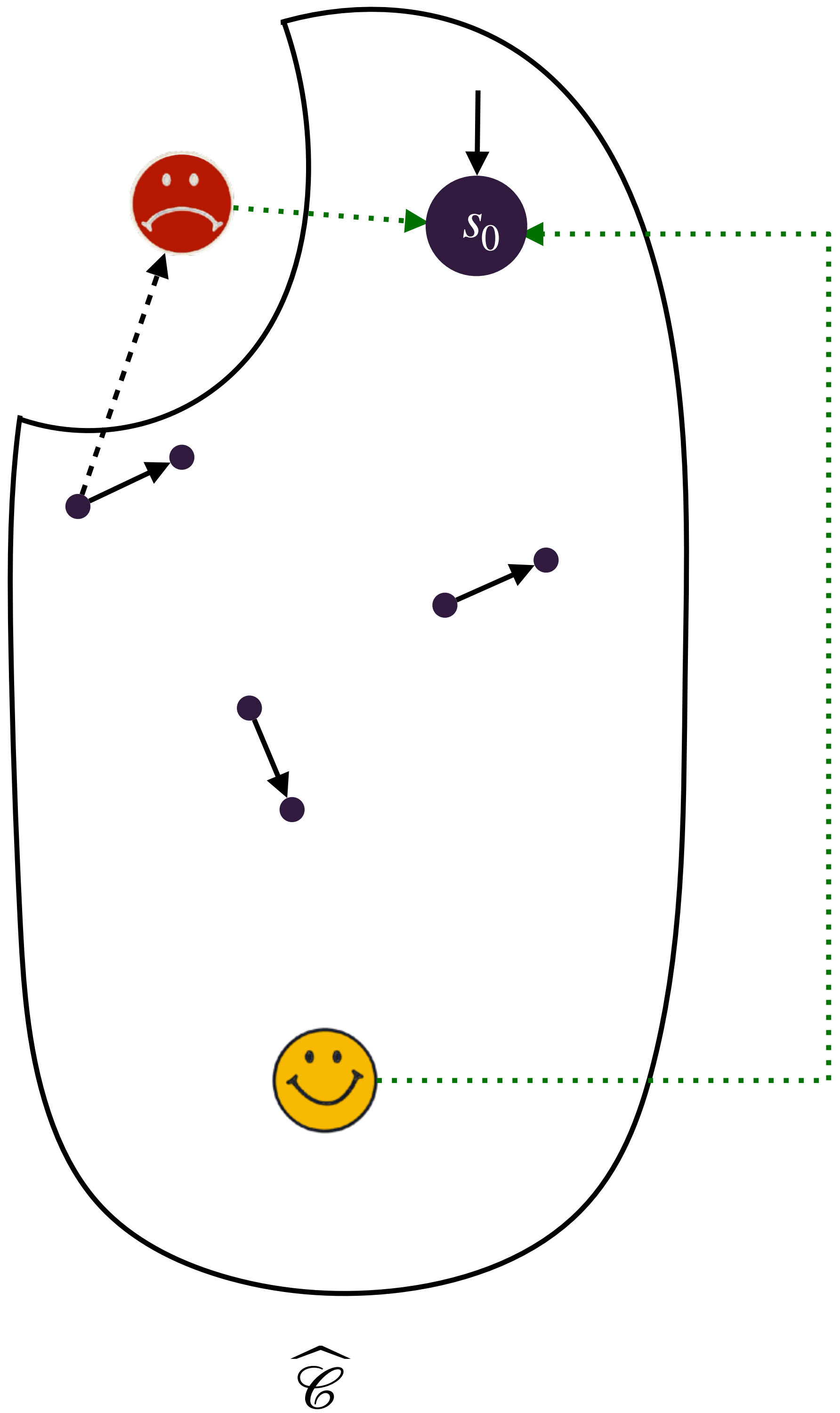

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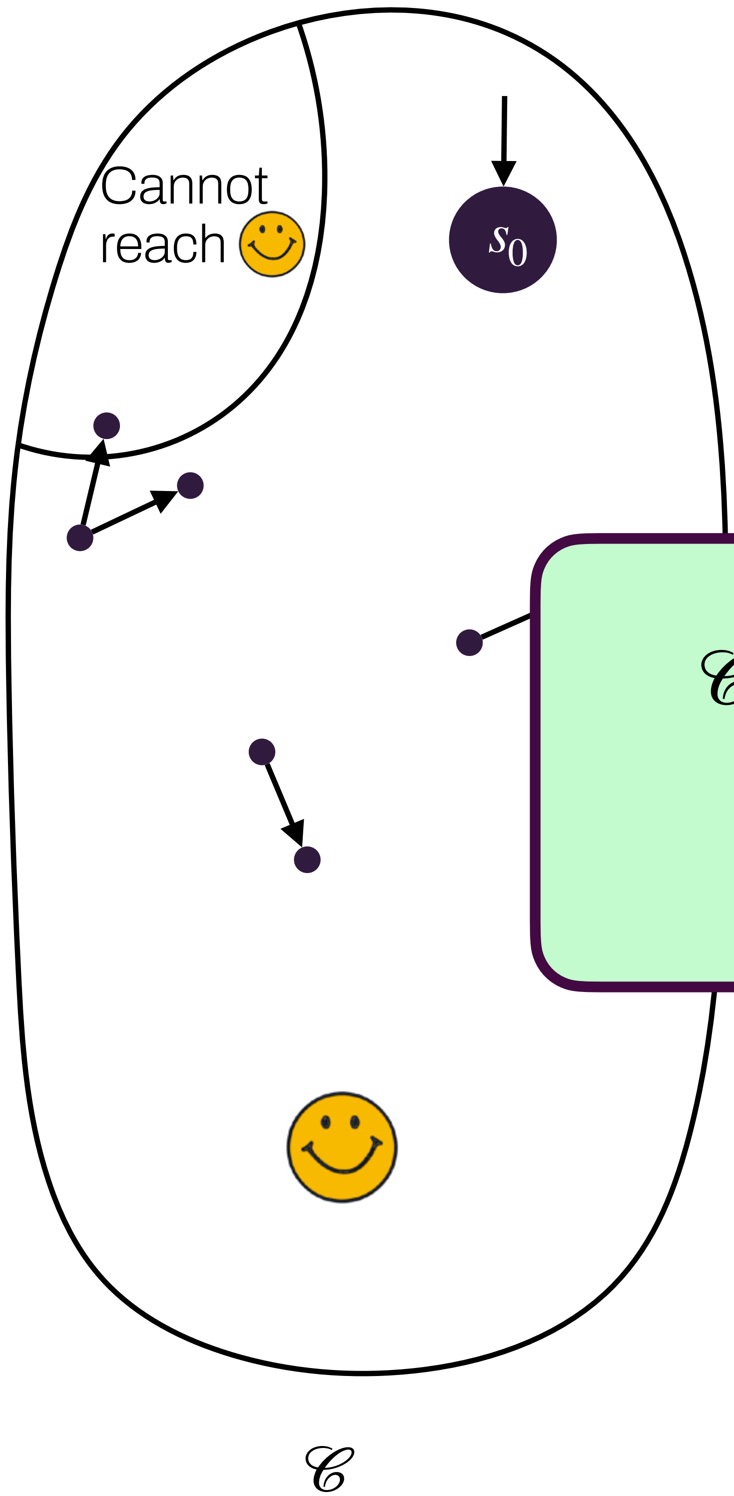
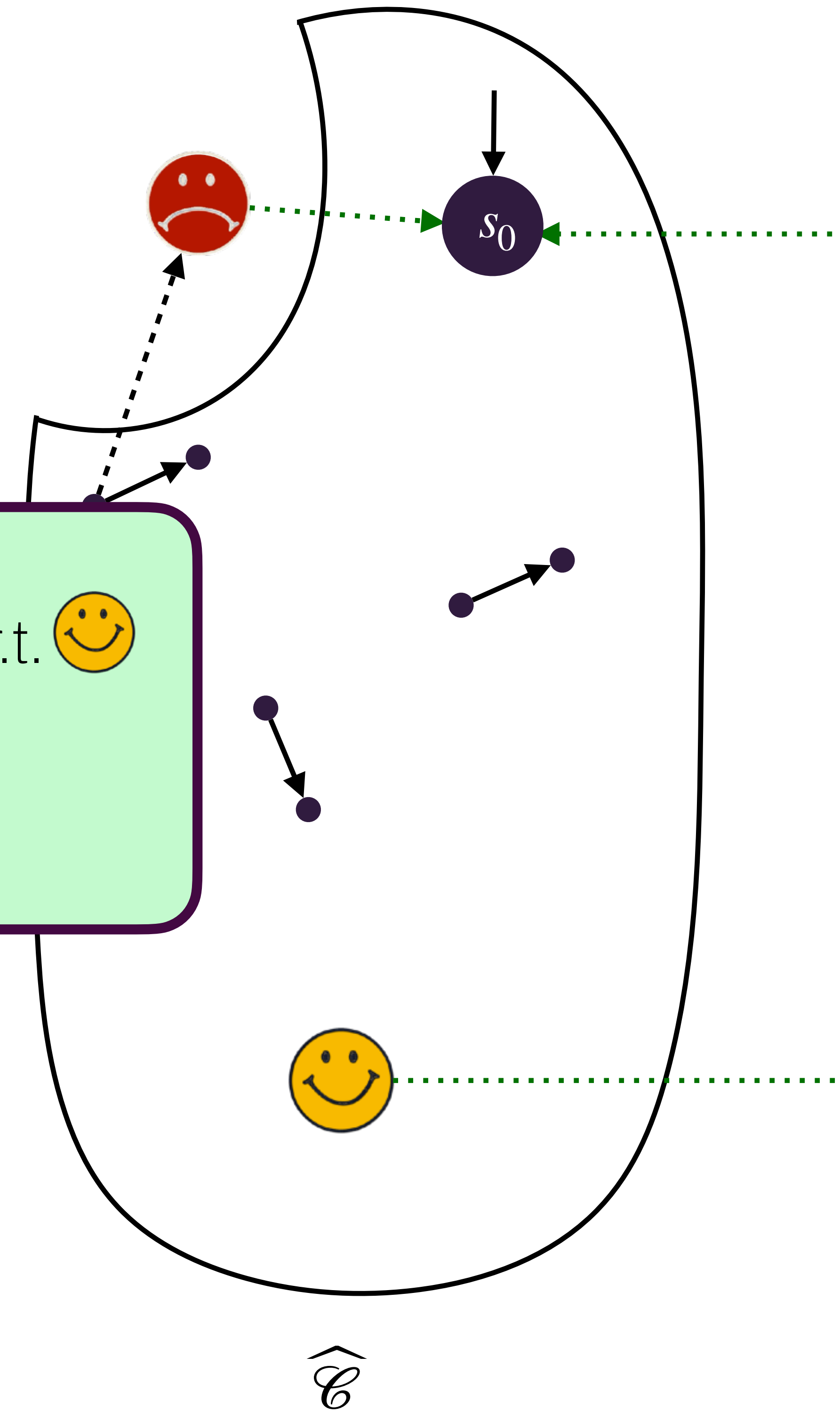


$\mathscr{C}$ is decisive from s_0 w.r.t. 😊
iff
$\widehat{\mathscr{C}}$ is recurrent



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Efficiency of sampling

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Termination and efficiency

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Efficiency of sampling

- $\mathscr{C}$ is decisive from s_0 w.r.t. 🙂 iff $\widehat{\mathscr{C}}$ is recurrent
- If $\widehat{\mathscr{C}}$ is positive recurrent, then sampling a single path in $\mathscr{C}$ will take finite time

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Efficiency of sampling

The time to sample even increases/diverges!

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Statistical guarantees

Hoeffding's inequalities

Let $\varepsilon, \delta > 0$, let $N \geq \frac{8}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

$$\mathbb{P}\left(\left| \frac{n_N}{N} - \mathbb{P}\left(\mathbf{F} \smiley \right) \right| \geq \frac{\varepsilon}{2} \right) \leq \delta$$

Statistical guarantees

Hoeffding's inequalities

Empirical average

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Statistical guarantees

Hoeffding's inequalities

Fix two parameters, the third one follows

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A slightly more general setting

- Given $L : S^+ \rightarrow \mathbb{R}$, the ☺-function $f_{L,☺}$ is $\mathbf{1}_{\mathbf{F}☺} \cdot L$
- We are interested in evaluating the quantity $\mathbb{E}(f_{L,☺})$
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The two previous approaches extend under the same conditions to B-bounded ☺-functions

Empirical estimation

Let $\varepsilon, \delta > 0$ s.t. $N \geq \frac{8B^2}{\varepsilon^2} \log\left(\frac{2}{\delta}\right)$. Then:

$$\mathbb{P}\left(\left|\frac{f_N}{N} - \mathbb{E}(f_{L,\smiley})\right| \geq \frac{\varepsilon}{2}\right) \leq \delta$$

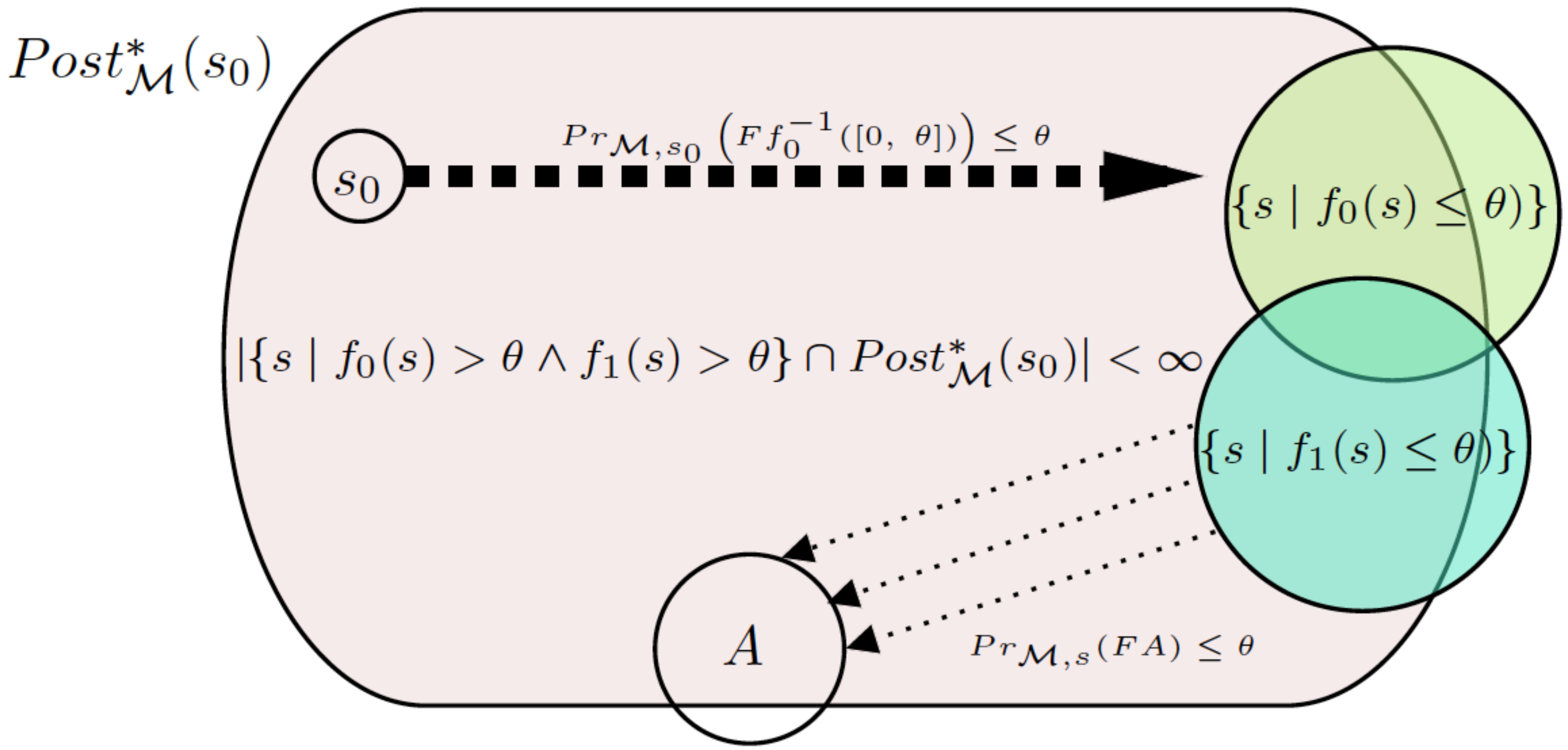
What can we do for
non-decisive Markov chains??

Another numerical generic approach

Divergent Markov Chains

A Markov chain $\mathcal{M}$ is *divergent* w.r.t. s_0 and A if there exist two computable functions f_0 and f_1 from S to $\mathbb{R}_{\geq 0}$ such that:

1. For all $0 < \theta < 1$, $\mathbf{Pr}_{\mathcal{M},s_0}(\mathbf{F} f_0^{-1}([0,\theta])) \leq \theta$;
2. For all $s \in S$, $\mathbf{Pr}_{\mathcal{M},s}(\mathbf{F} A) \leq f_1(s)$;
3. For all $0 < \theta < 1$, $\{s \mid f_0(s) > \theta \wedge f_1(s) > \theta\} \cap Post^*_{\mathcal{M}}(s_0)$ is finite.



[FHY23b] A. Finkel, S. Haddad, L. Ye. Introducing divergence for infinite probabilistic models (RP'23)

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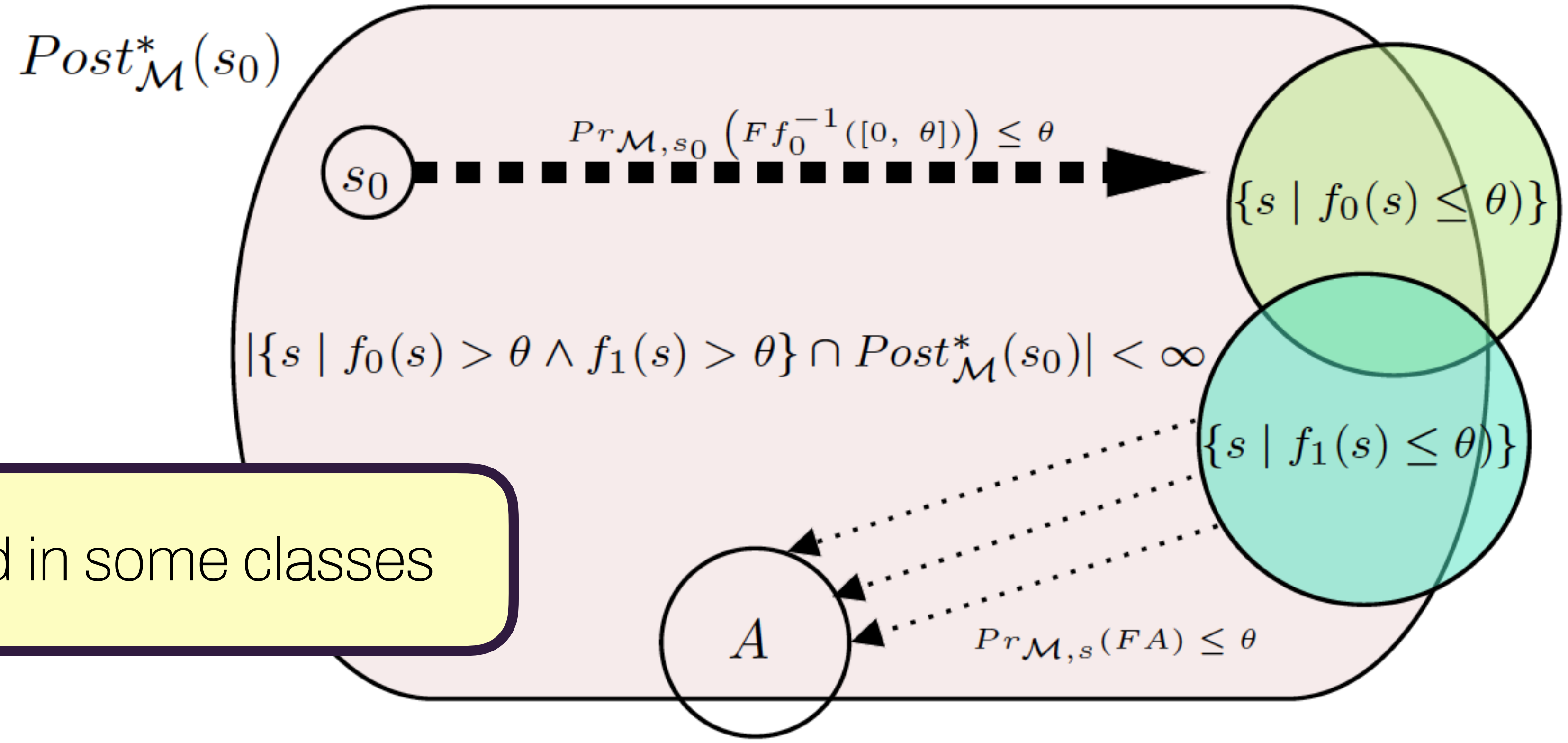

Can be decided in some classes

[FHY23b] A. Finkel, S. Haddad, L. Ye. Introducing divergence for infinite probabilistic models (RP'23)

Importance sampling for rare events evaluation

- Issue: rare events in $\mathscr{C}$

Rare-Event Problem for Statistical Model Checking

Problem Statement

- We want to estimate the probability of a rare event e occurring with probability close to 10^{-15}.
- We want a *confidence level* of 0.99.
- We are able to compute 10^9 trajectories.

Possible Outcomes

Number of occurrences of e	Probability	Confidence interval
0	$\approx 1 - 10^{-6}$	$[0, 7.03 \cdot 10^{-9}]$
1	$\leq 10^{-6}$	$[6.83 \cdot 10^{-10}, 1.69 \cdot 10^{-9}]$
$n > 1$	$\leq 10^{-12}$	$> 6.83 \cdot 10^{-10}$

[Bar14] B. Barbot. Acceleration for statistical model checking (PhD thesis)
[BHP12] B. Barbot, S. Haddad, C. Picaronny. Coupling and Importance Sampling for Statistical Model Checking (TACAS'12)

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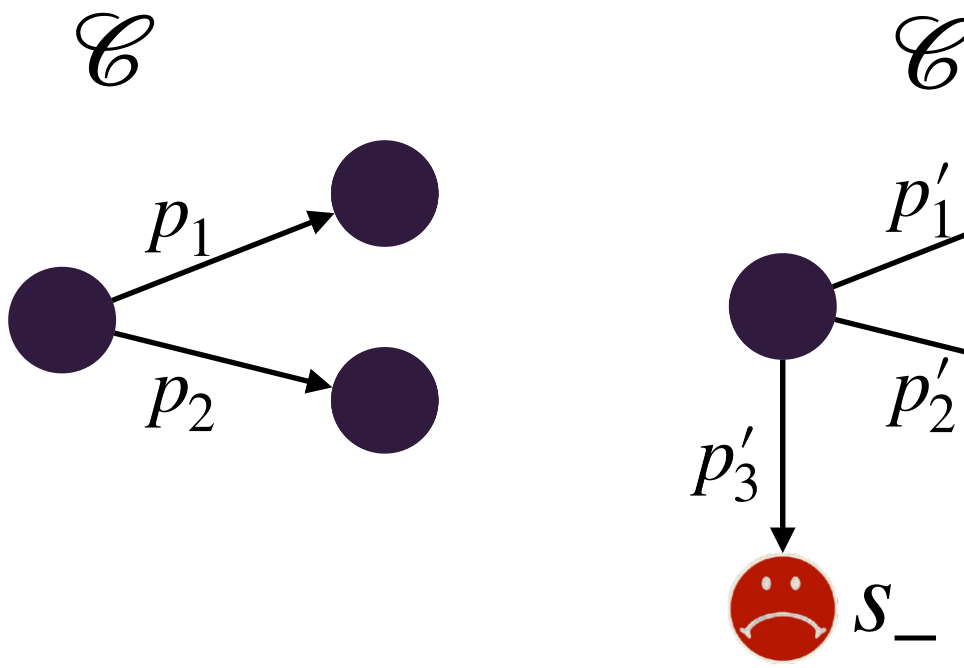

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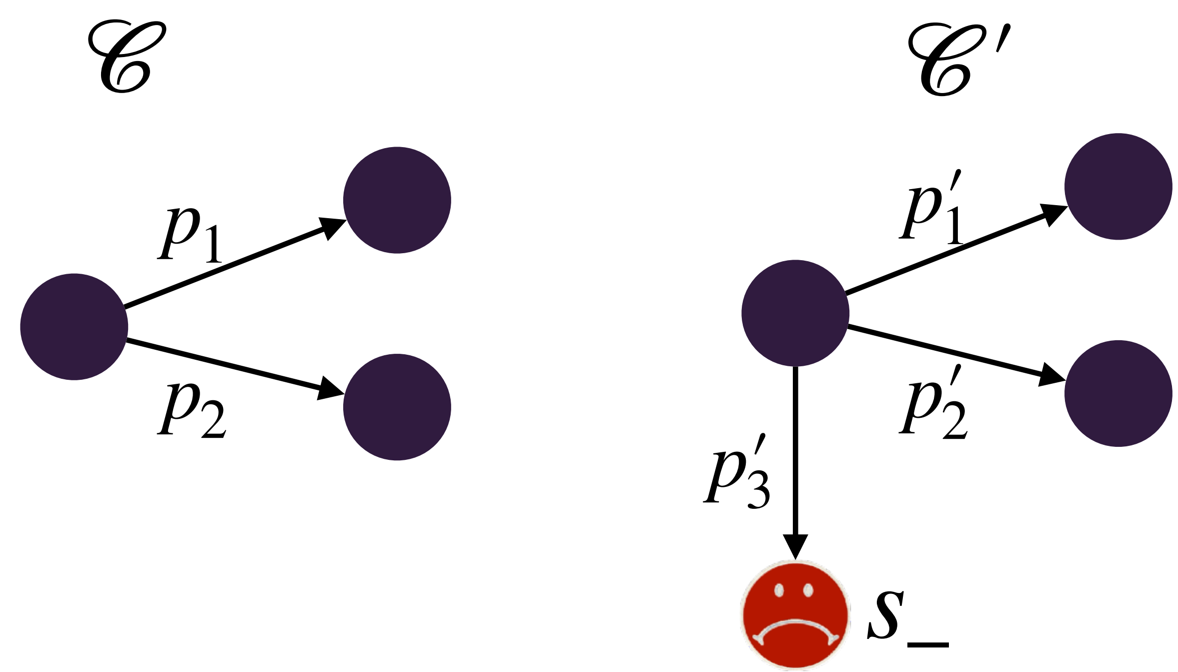
$$\gamma(\rho) = \begin{cases} \dfrac{P(\rho)}{P'(\rho)} & \text{if } \rho \text{ ends in } 🙂 \\ 0 & \text{otherwise} \end{cases}$$

$$L' = L \cdot \gamma$$



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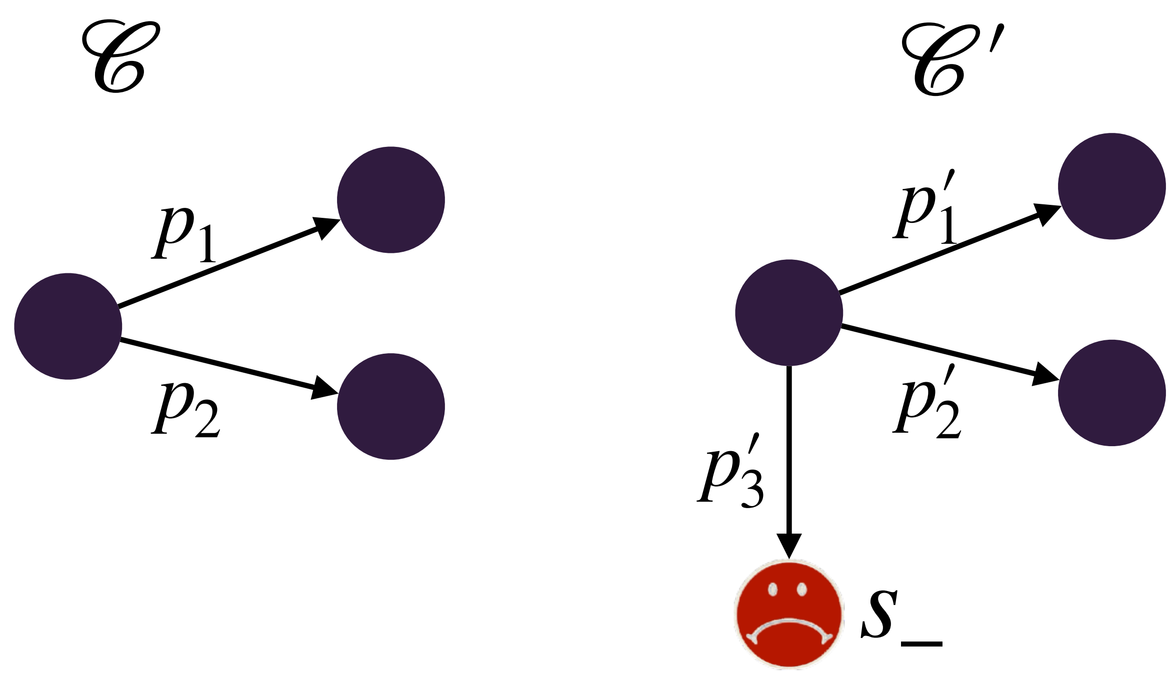

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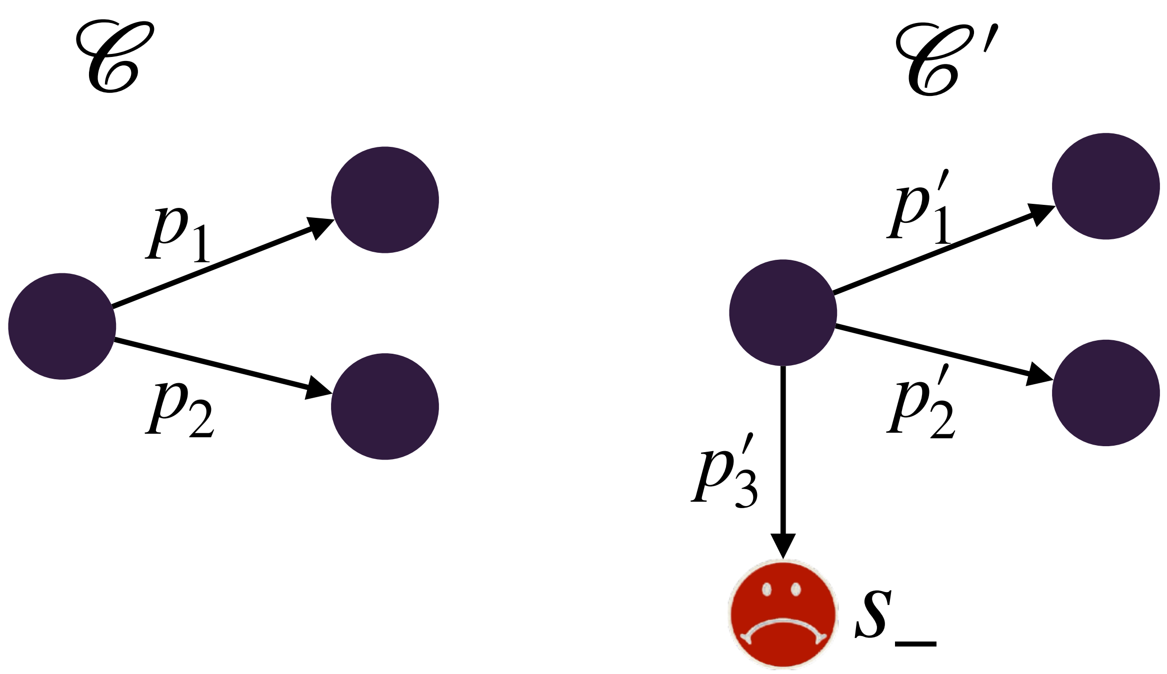

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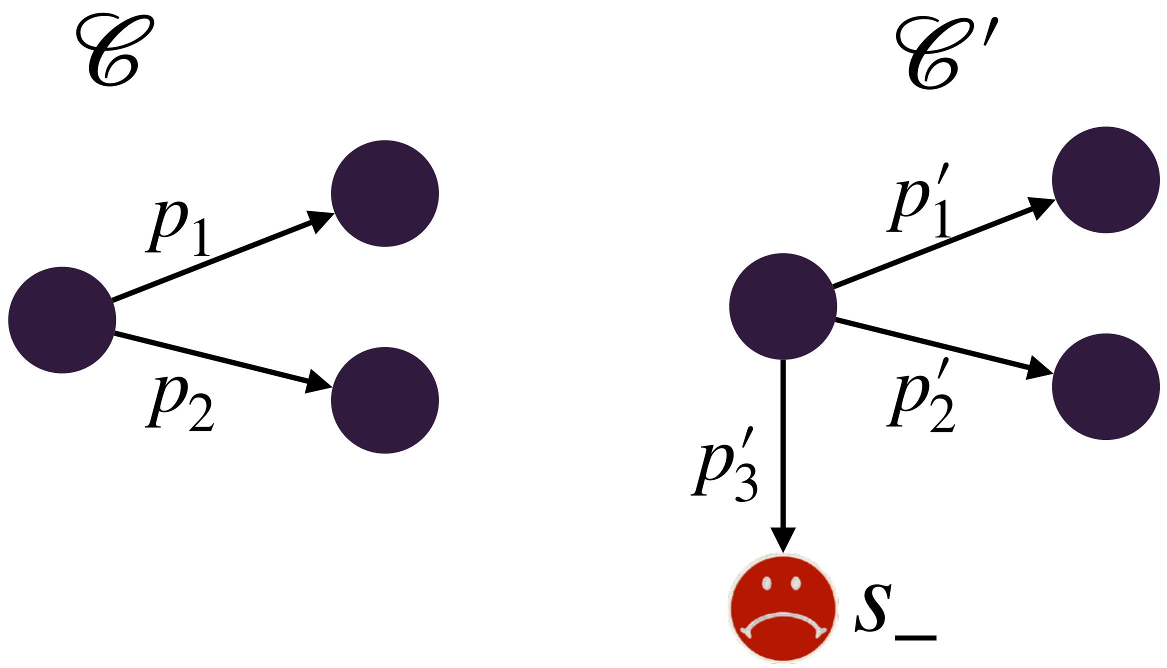

+ setting giving statistical guarantees

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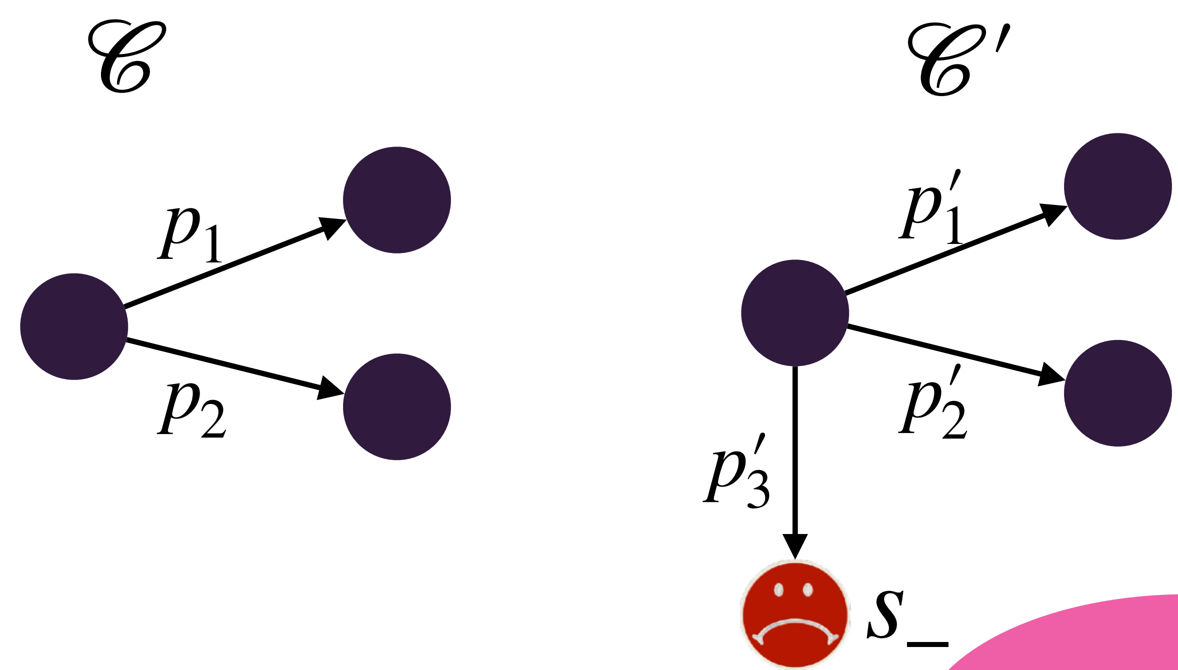

Before that, only estimators!!

+ setting giving statistical guarantees

[Bar14] B. Barbot. Acceleration for statistical model checking (PhD thesis)
[BHP12] B. Barbot, S. Haddad, C. Picaronny. Coupling and Importance Sampling for Statistical Model Checking (TACAS'12)

We propose to use the importance sampling approach to analyze some non-decisive DTMCs!

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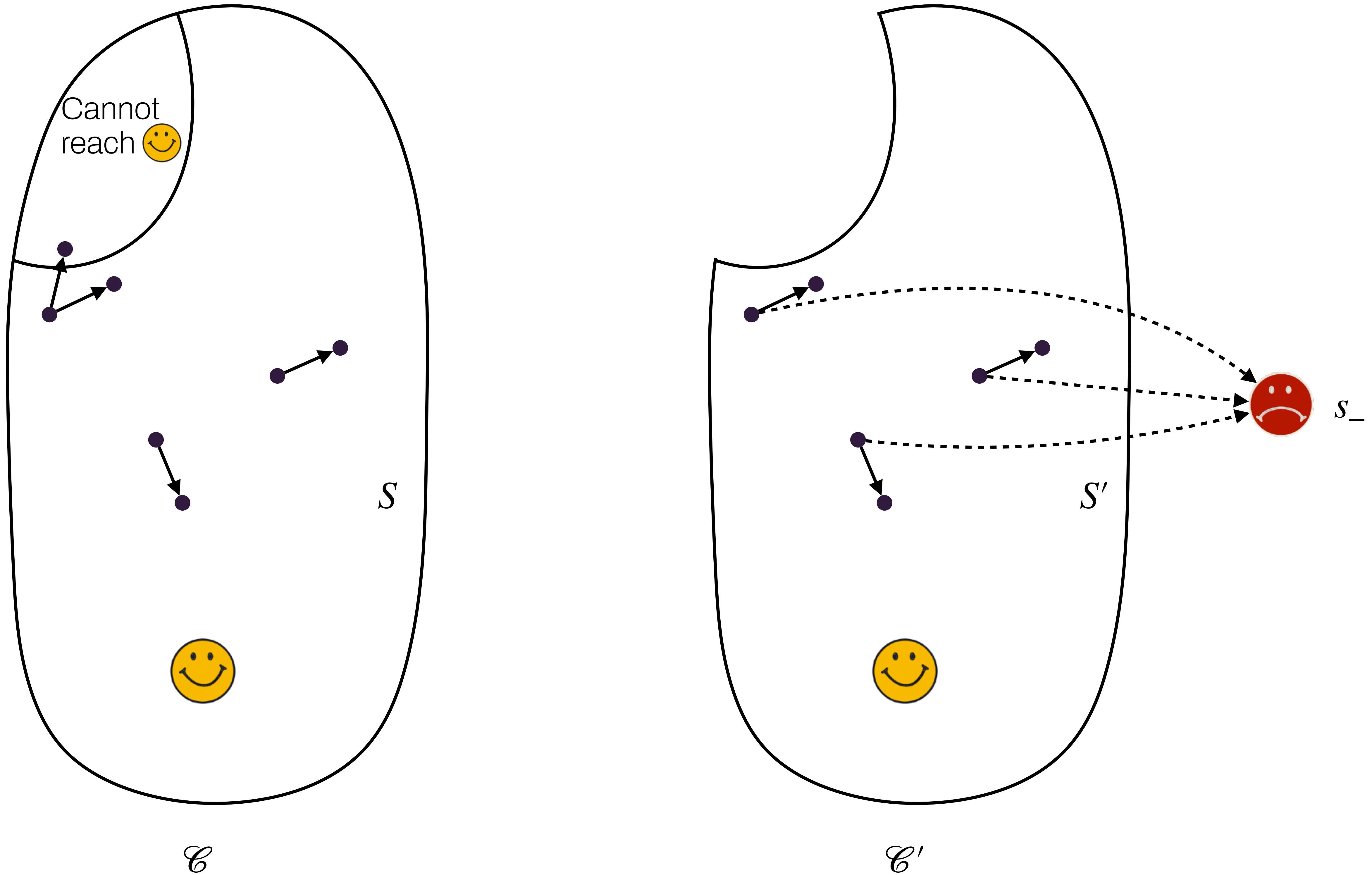
First time that importance sampling is used not to accelerate the analysis, but to enable the analysis

Biased Markov chain



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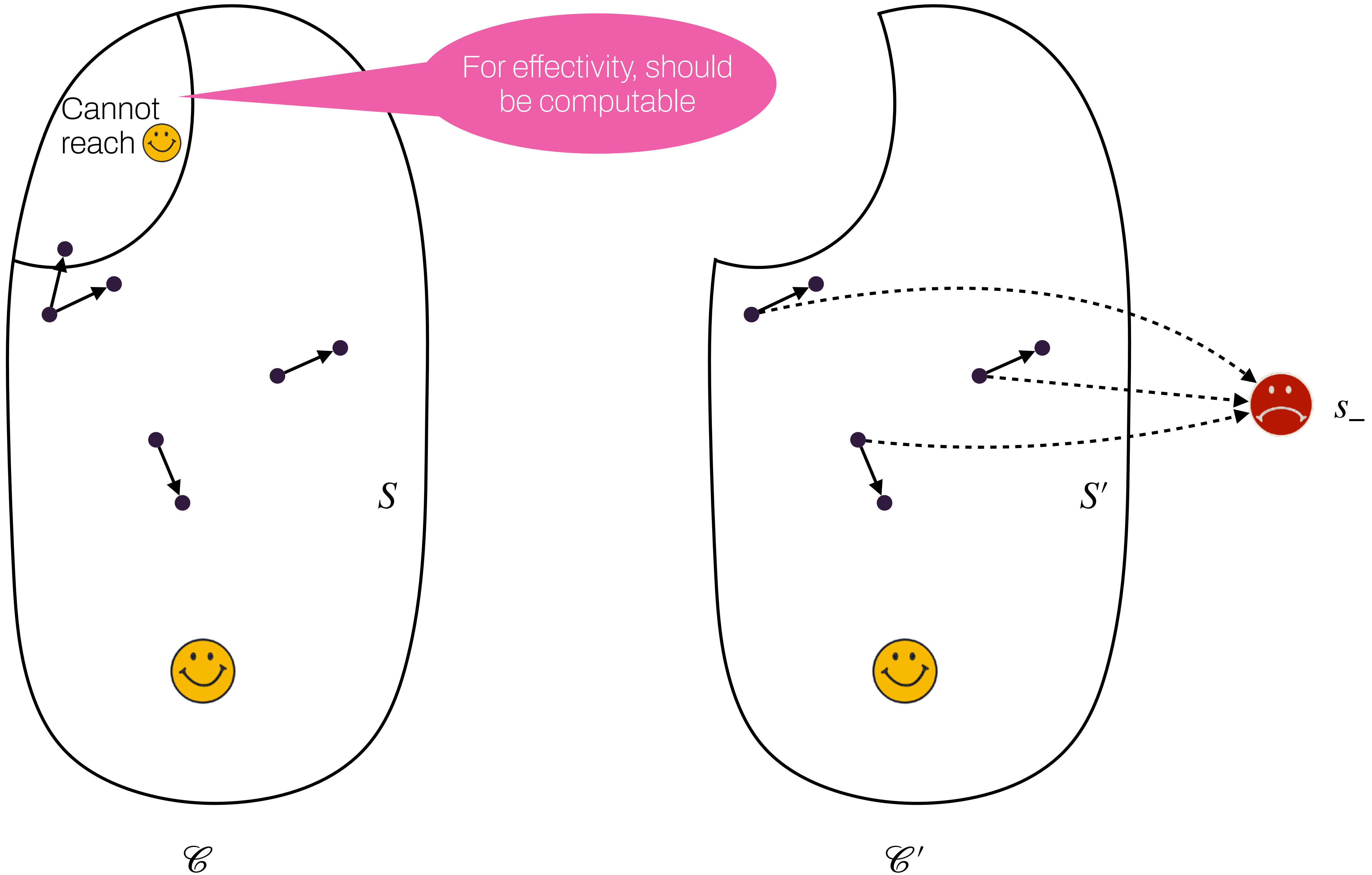

Properties of the biased Markov chain

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[BHP12] *B. Barbot, S. Haddad, C. Picaronny. Coupling and Importance Sampling for Statistical Model Checking (TACAS'12)*

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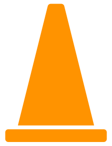
  - L' can be unbounded even if L is bounded 

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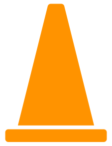
  - L' can be unbounded even if L is bounded 
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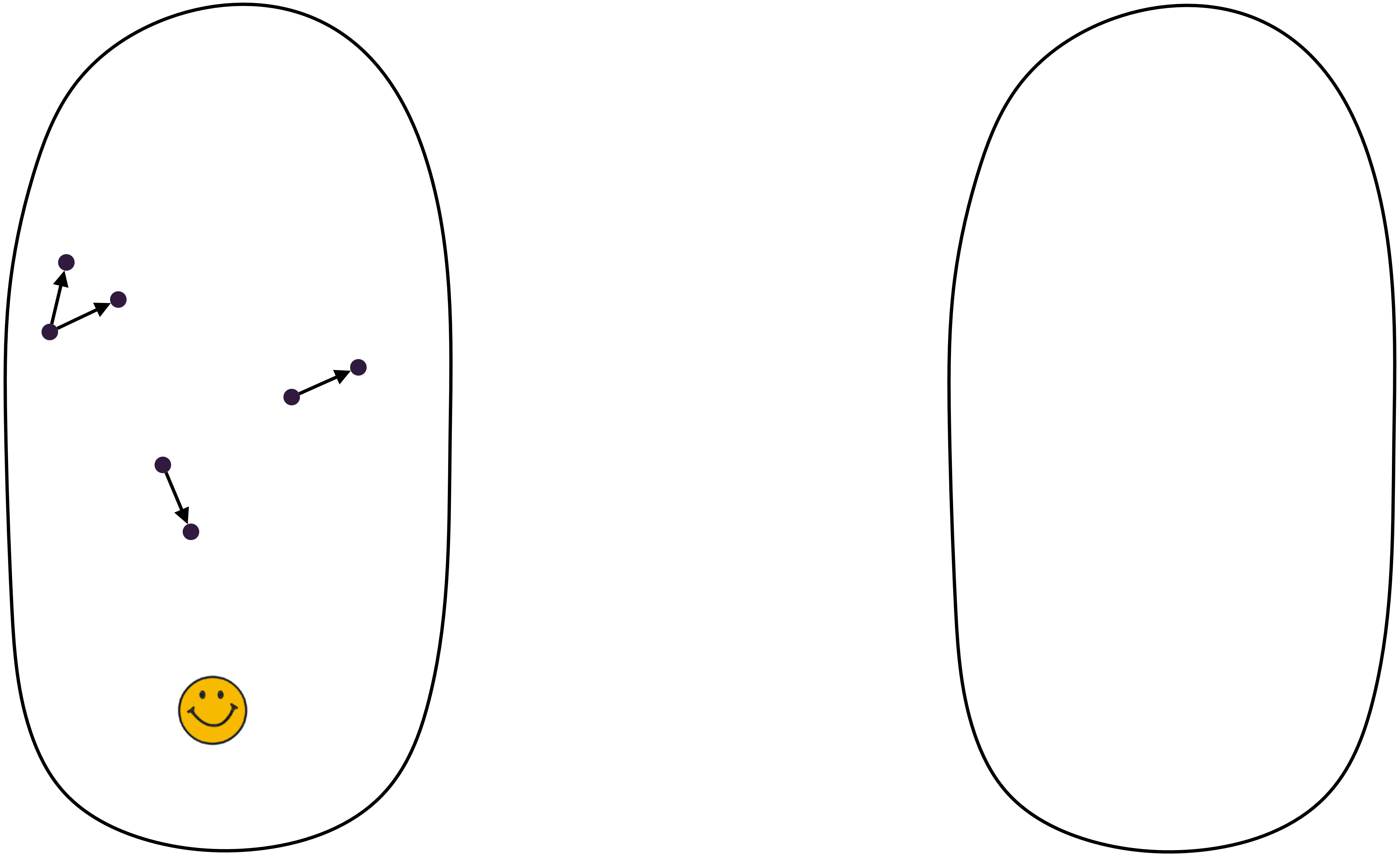
  - [BHP12] for rare events: approach for finite Markov chains via coupling and abstractions with reduced variance

[BHP12] B. Barbot, S. Haddad, C. Picaronny. Coupling and Importance Sampling for Statistical Model Checking (TACAS'12)

Our approach



$\mathscr{C}$

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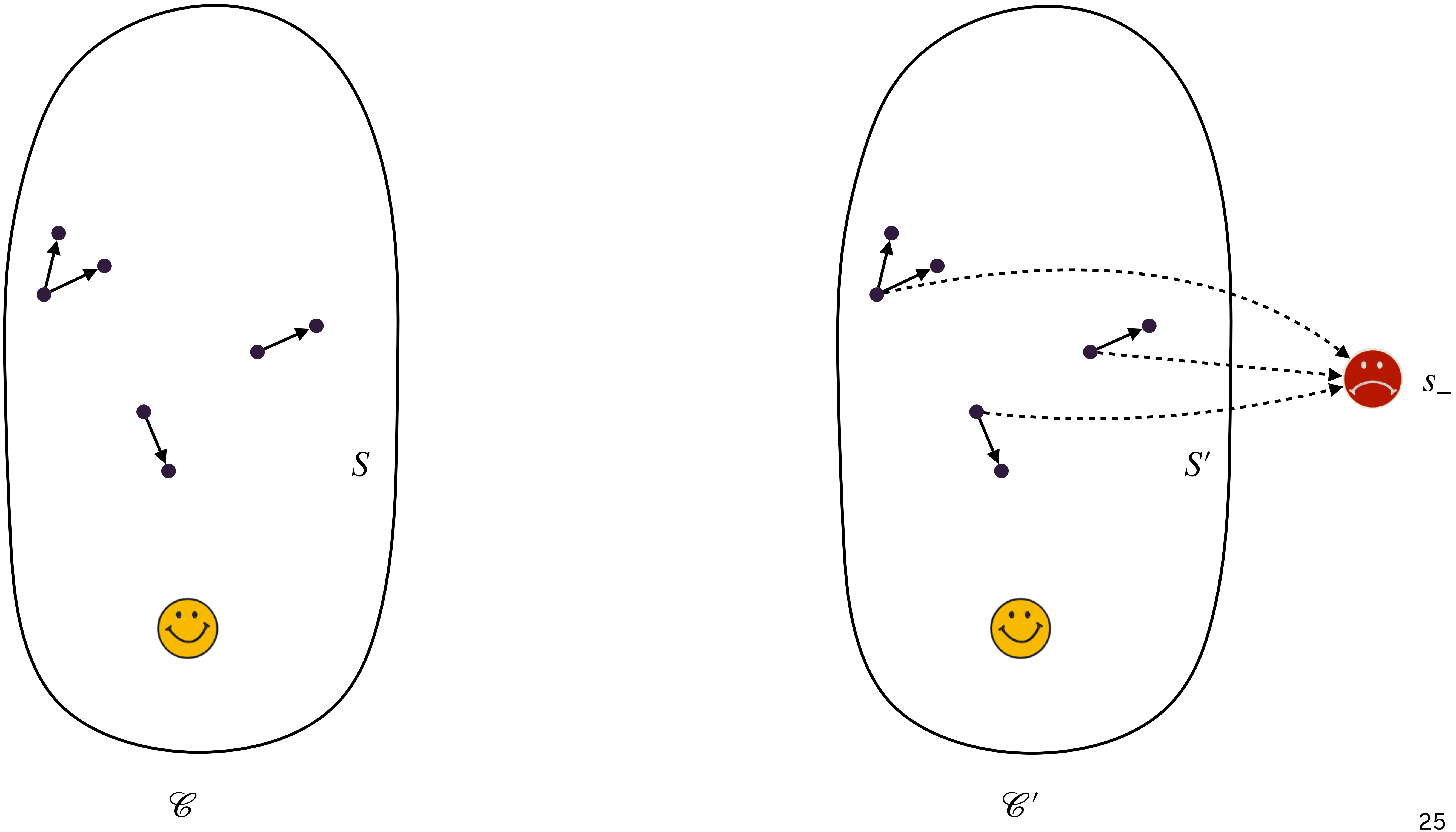

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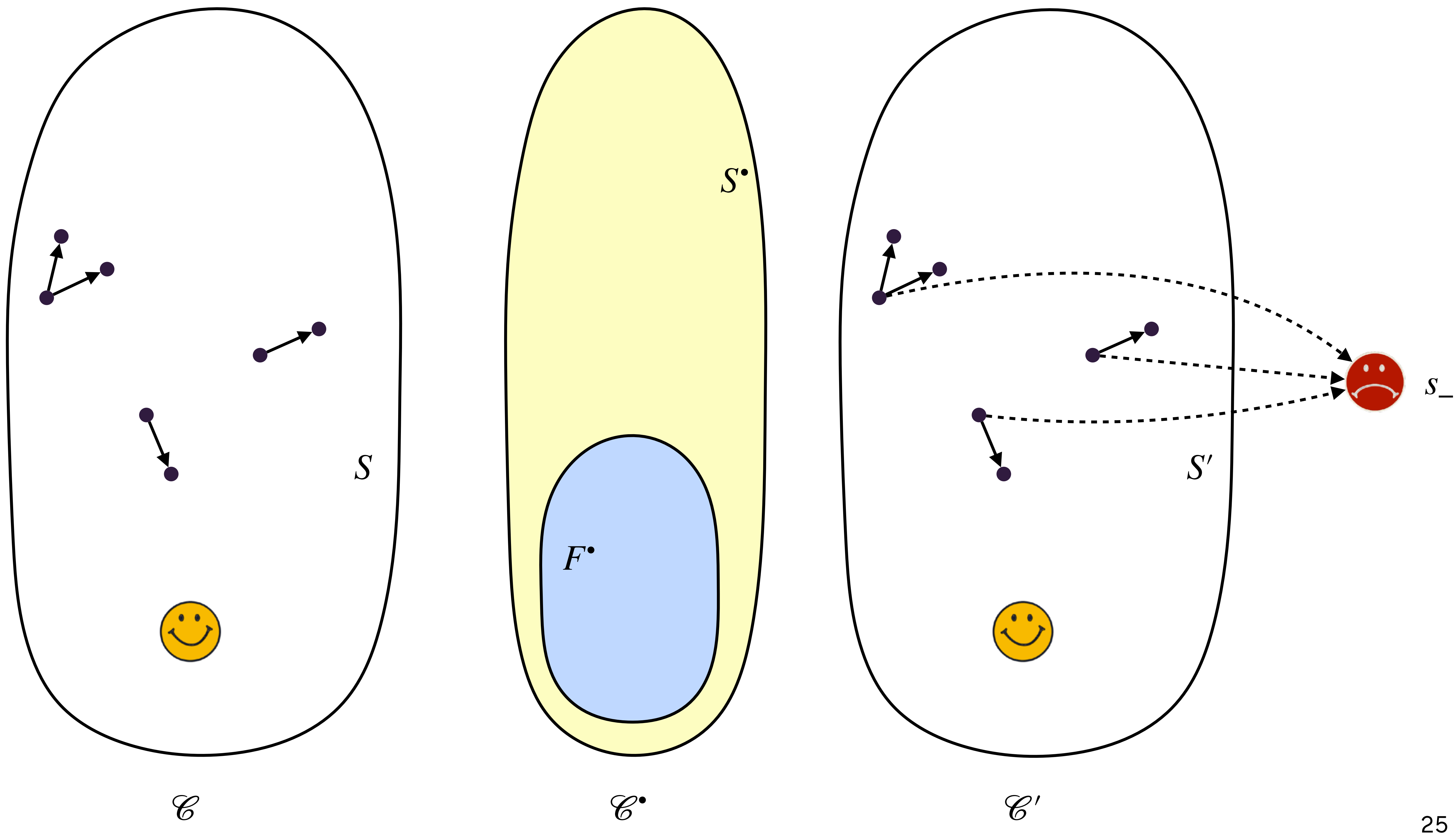

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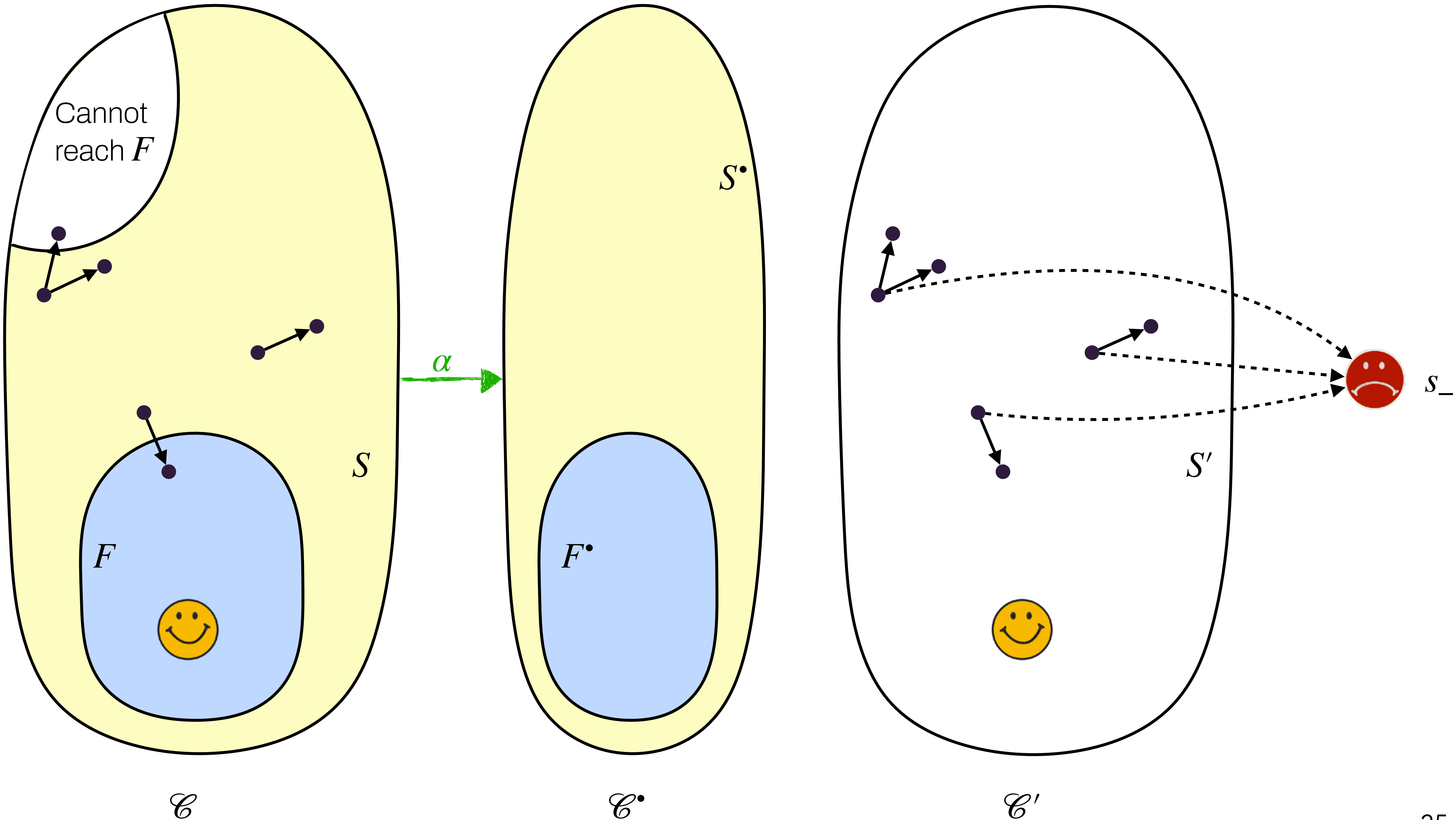

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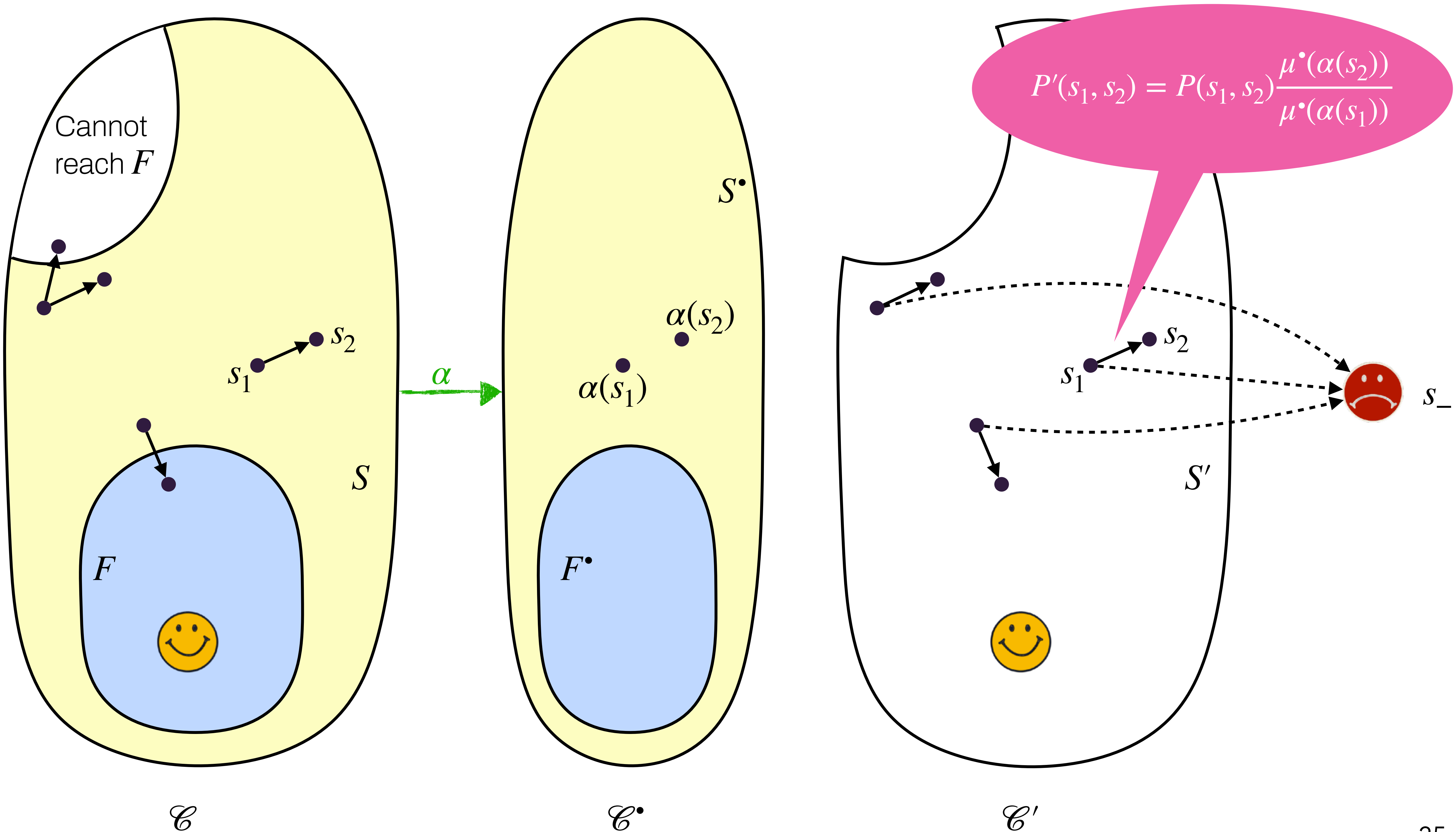

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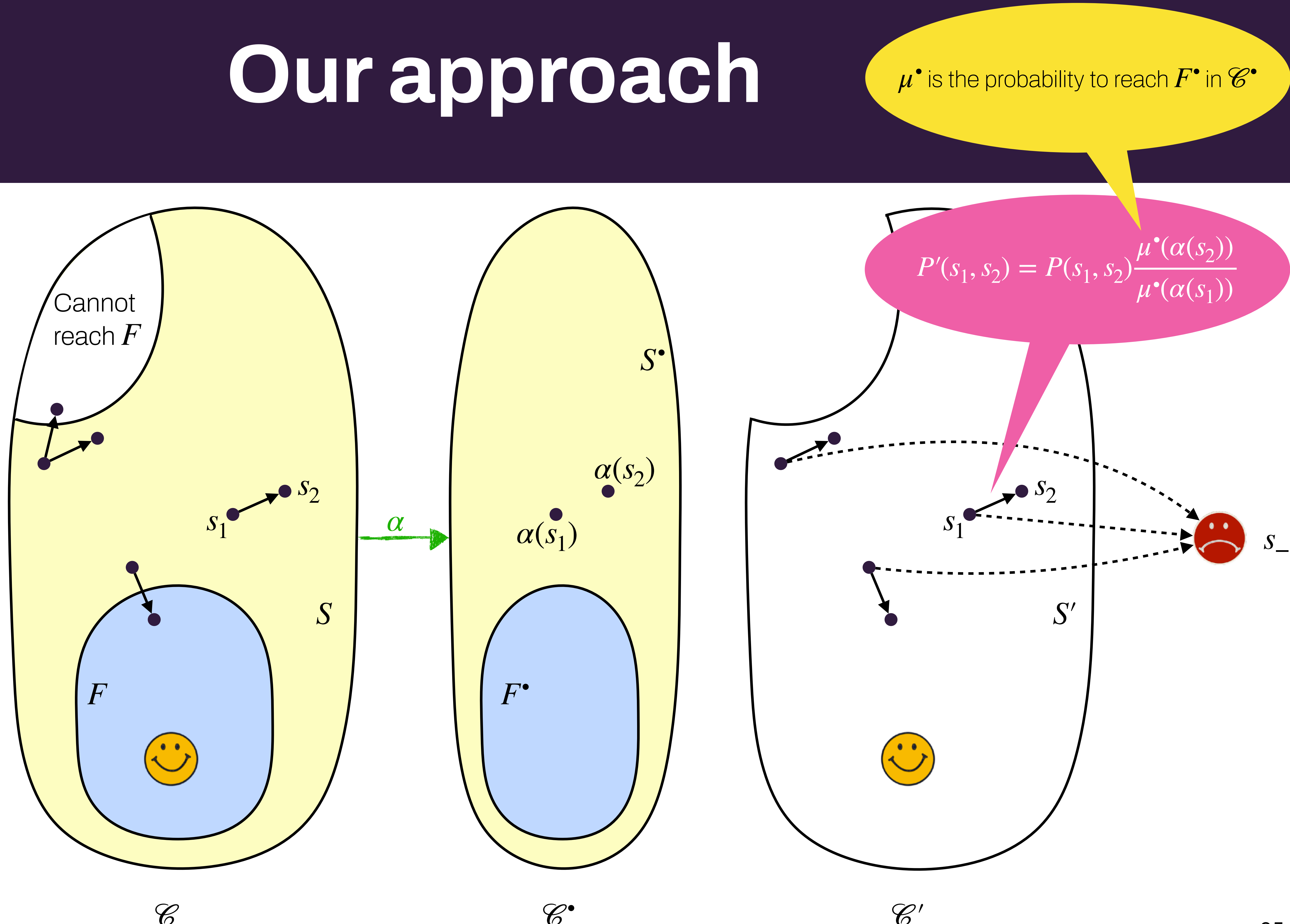

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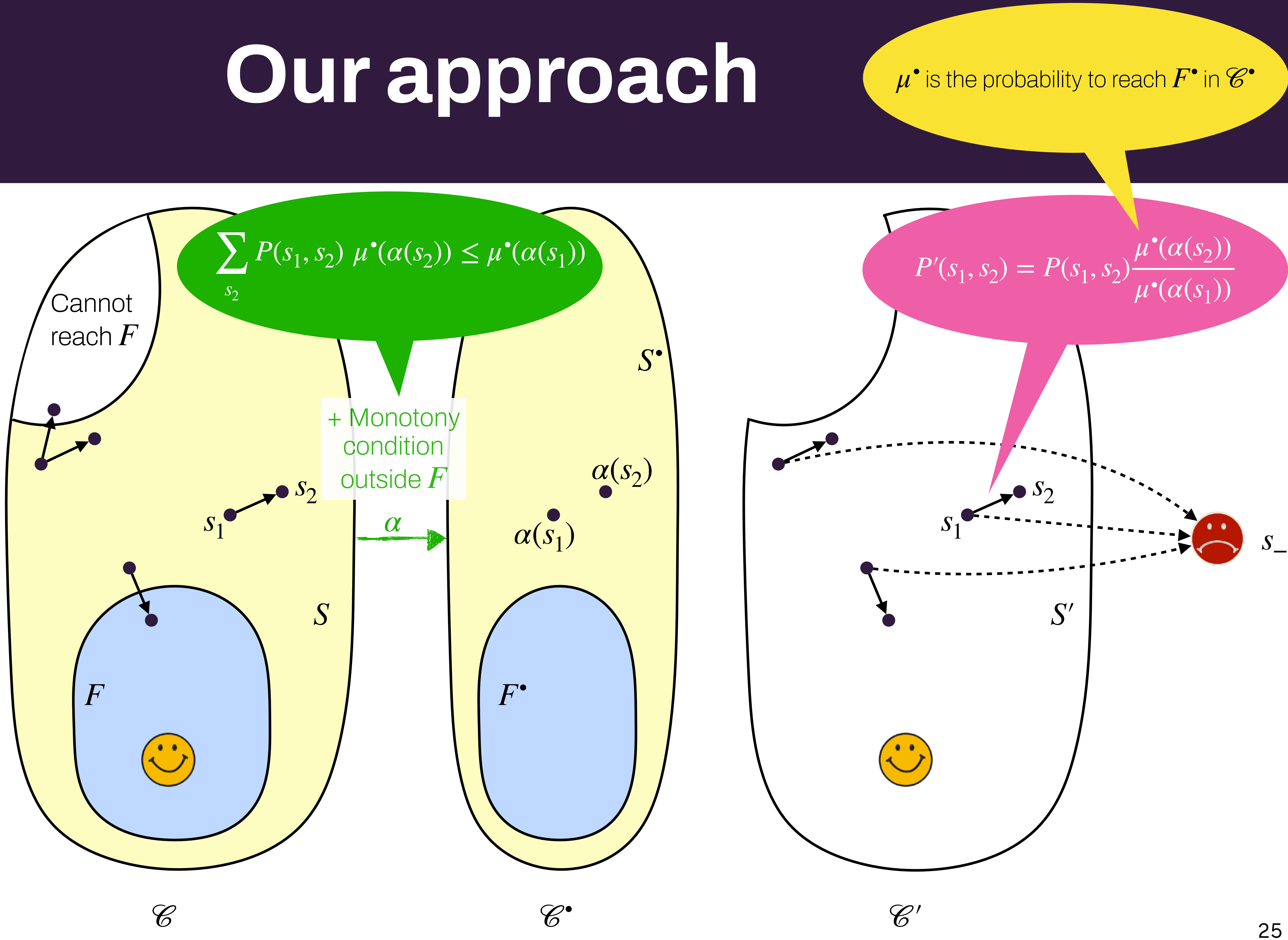

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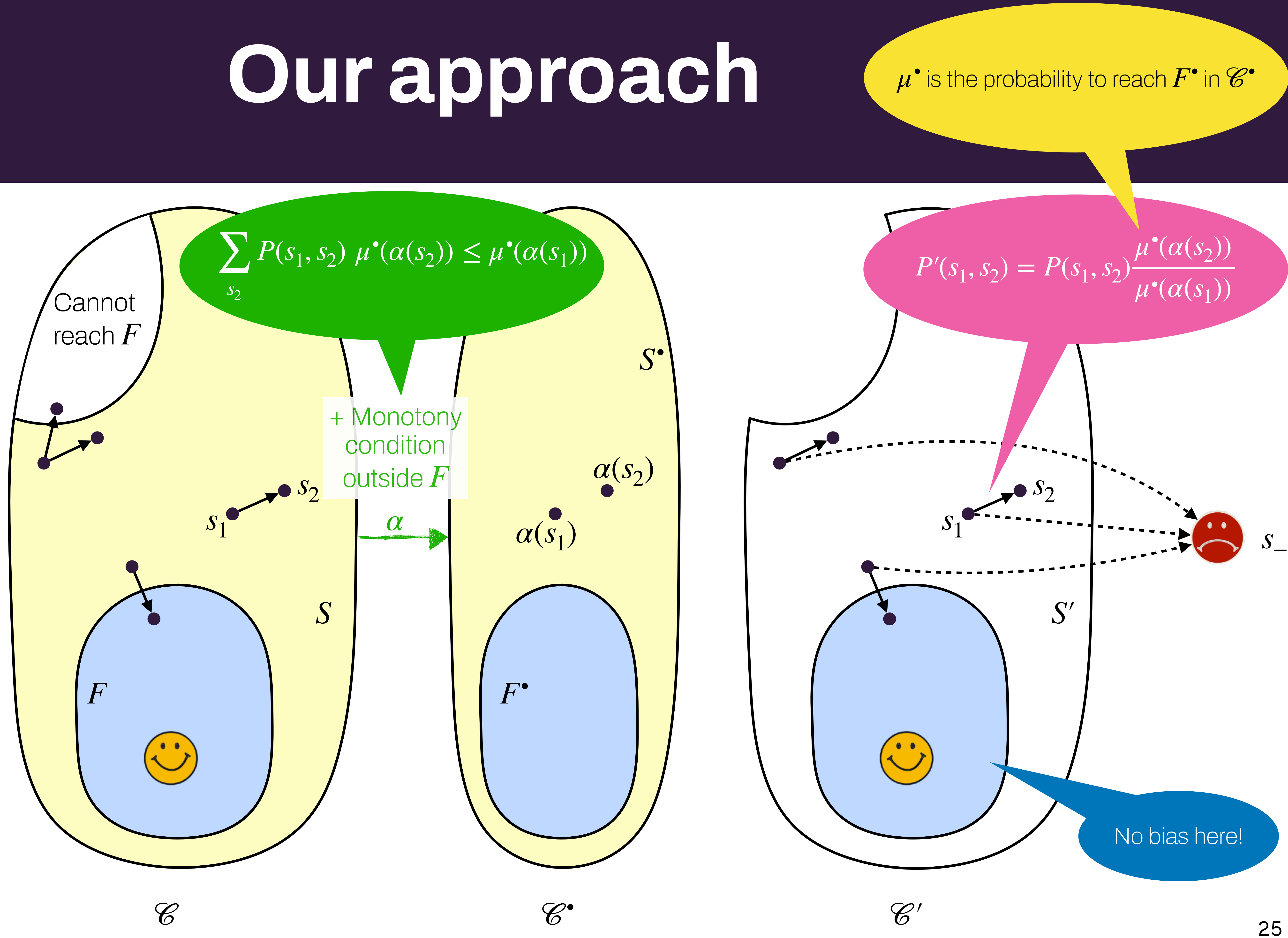

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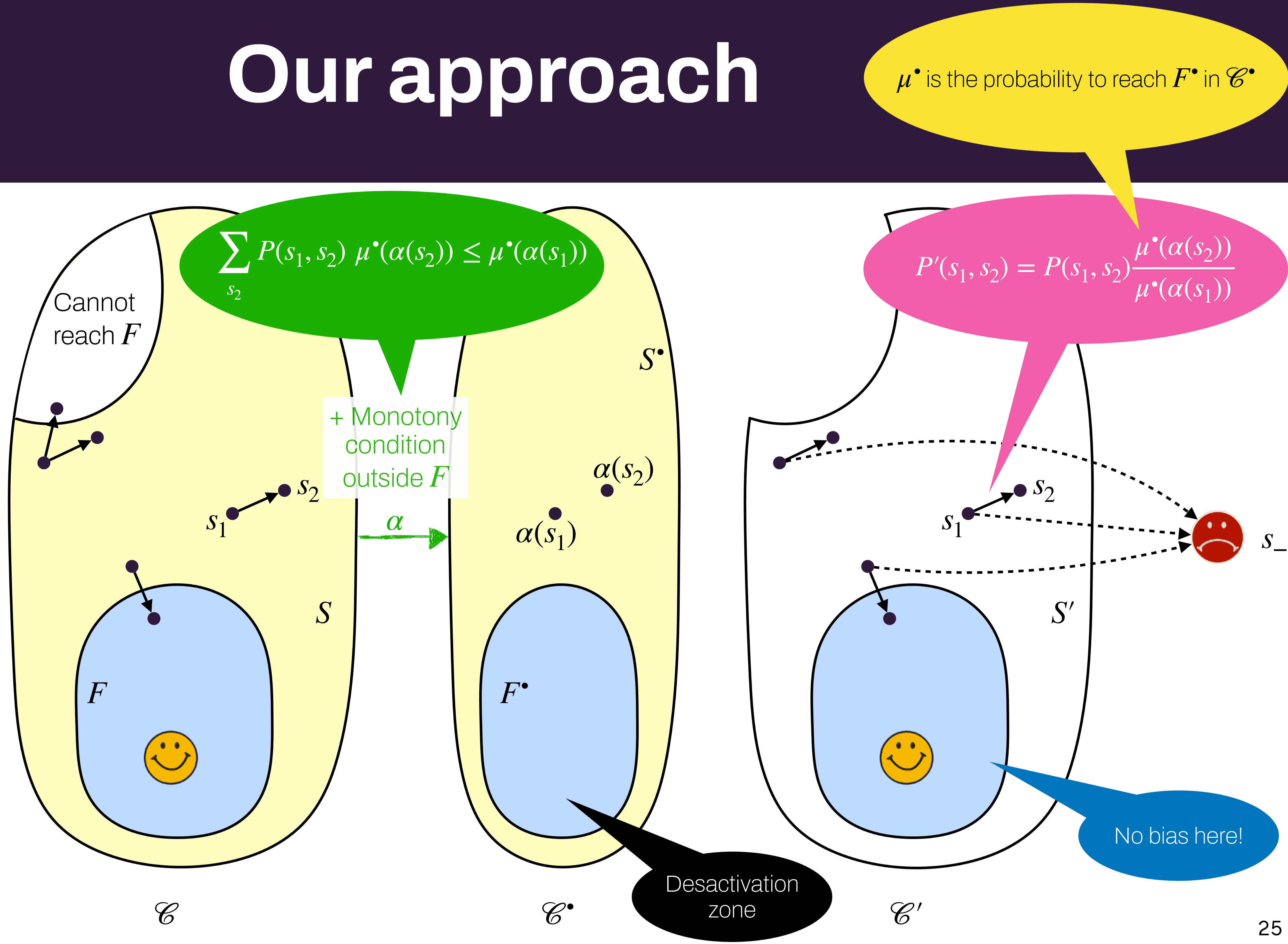

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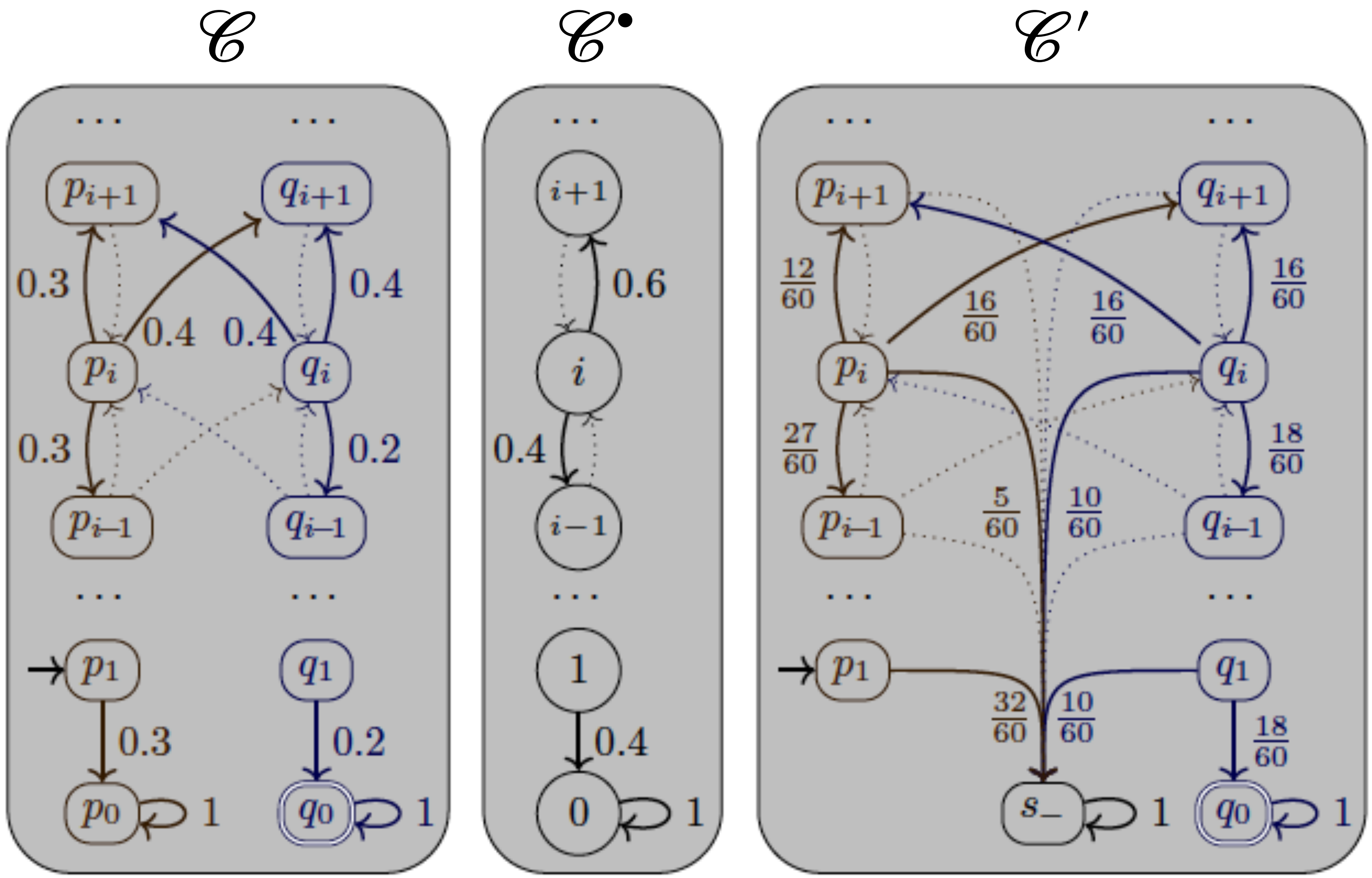
  - To compute $\mu^{\bullet}(\,\cdot\,)$ (useful in two places: to sample paths and to compute the final value when hitting ☺)

Role of F

- Standard approach for importance sampling: no set F
  (F coincides with )
- Will be useful to adjust the properties satisfied by the abstraction to be correct
  - Requirement will be « outside F »
  - For instance, congestion of systems

Example



$F = \{$  $\} = \{q_0\}$

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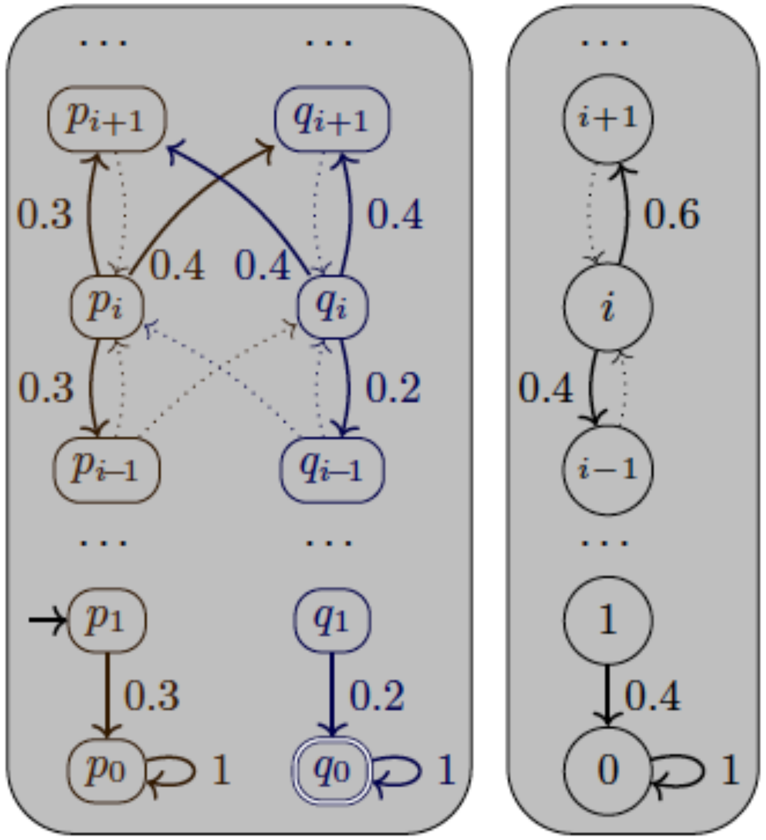

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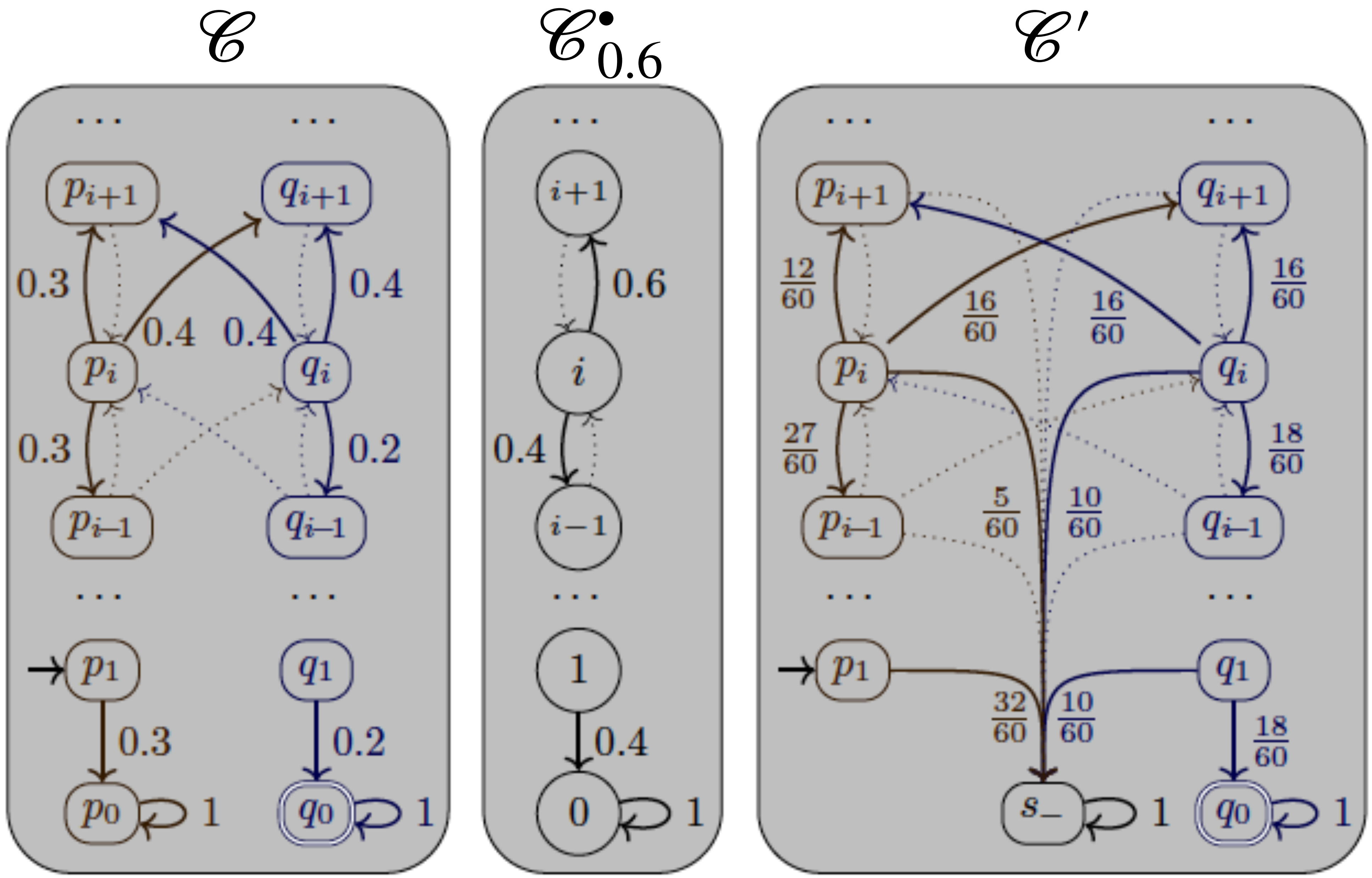
We lose finite time sampling in general

Reached almost-surely

+ slight generalization via « uniformisation »

Example



$\mathscr{C}$ is not decisive

$\mathscr{C}'$ is decisive

Application to probabilistic pushdown automata

[EKM06] J. Esparza, A. Kucera, R. Mayr. Model checking pushdown automatas (LMCS)

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Analysis can be done using the first-order theory of the reals

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Application to probabilistic pushdown automata

Analysis can be done using the first-order theory of the reals

- Automaton with a stack
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- Can be seen as a layered Markov chain, using the length of the stack content

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Implementation

https://cosmos.lacl.fr/
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- Some experiments have been done

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The larger is p, the larger is N_0

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  - Use the abstraction $\mathscr{C}_p^\bullet$ with desactivation zone $F^\bullet = [0; N_0]$

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- p big: large desactivation zone (N_0)
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What's the trade-off?

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The larger is p, the larger is N_0

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What's the trade-off?

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Note: in all experiments, the confidence is set to $99\ \%$

First example

- State-free proba. pushdown automaton $\mathscr{C}$
  $A \xrightarrow{1} C \quad A \xrightarrow{n} BB \quad B \xrightarrow{5} \varepsilon$
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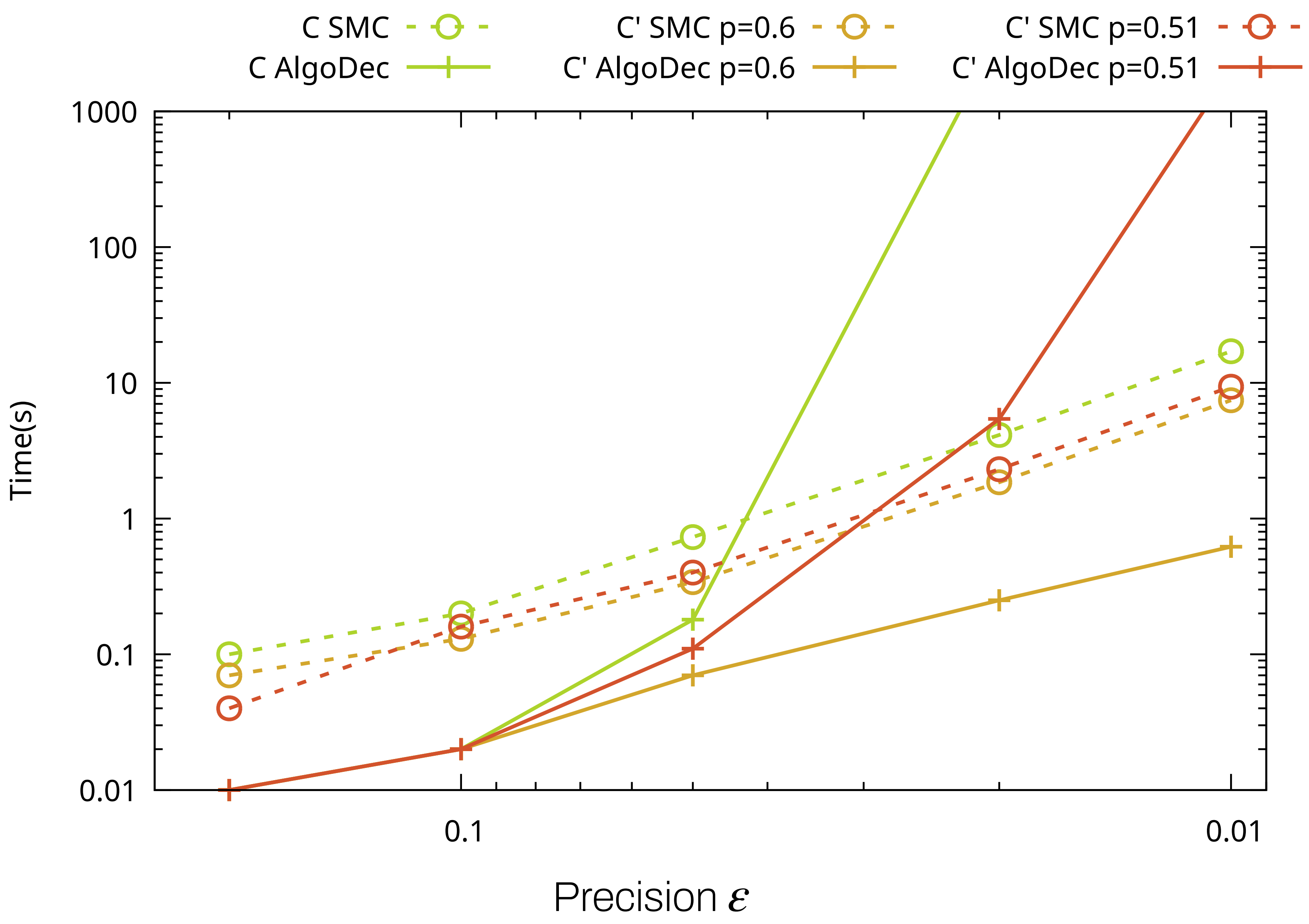
- It is decisive
- It is p-divergent for every $1/2 < p < 1$

First example
Experimental results



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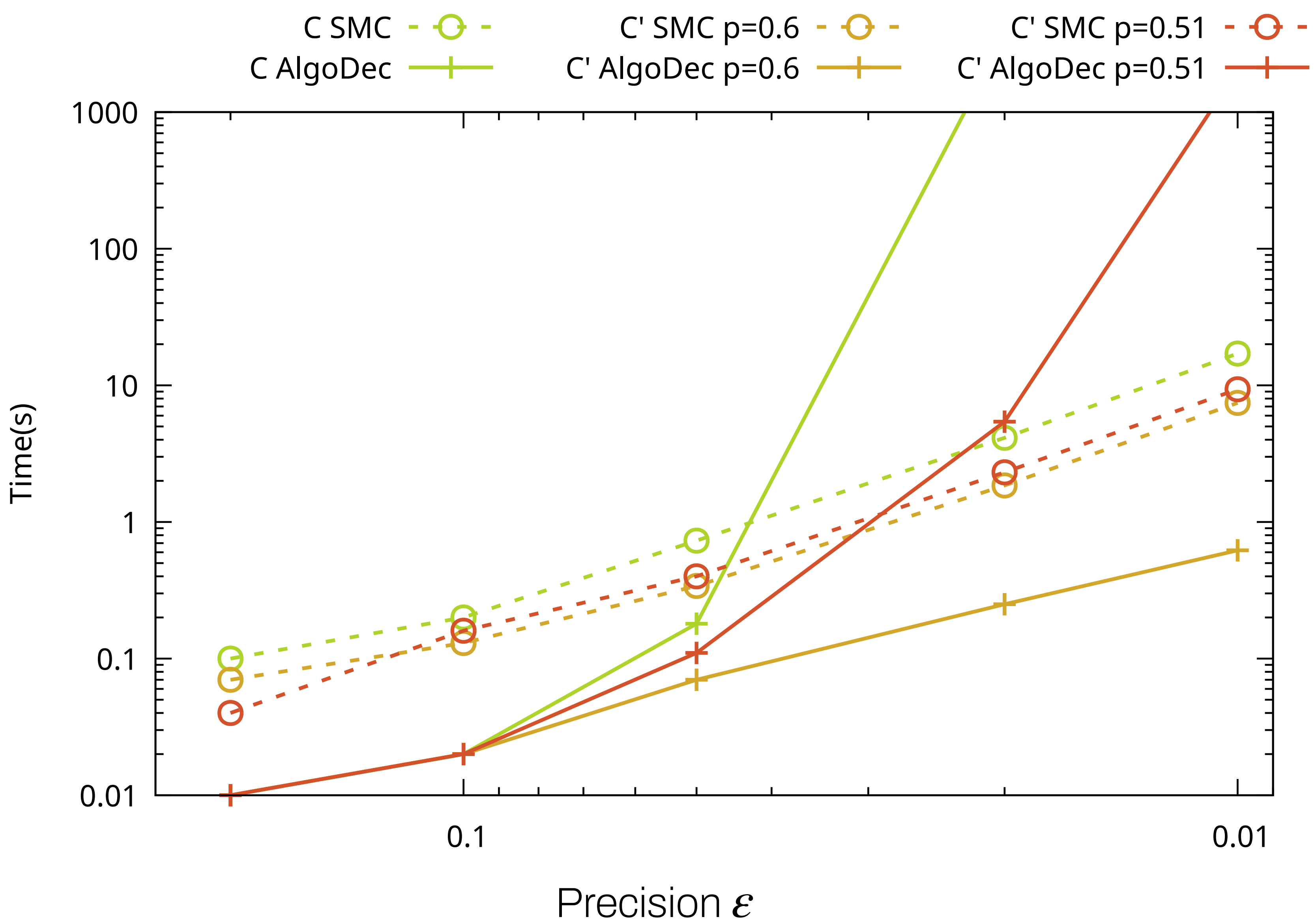

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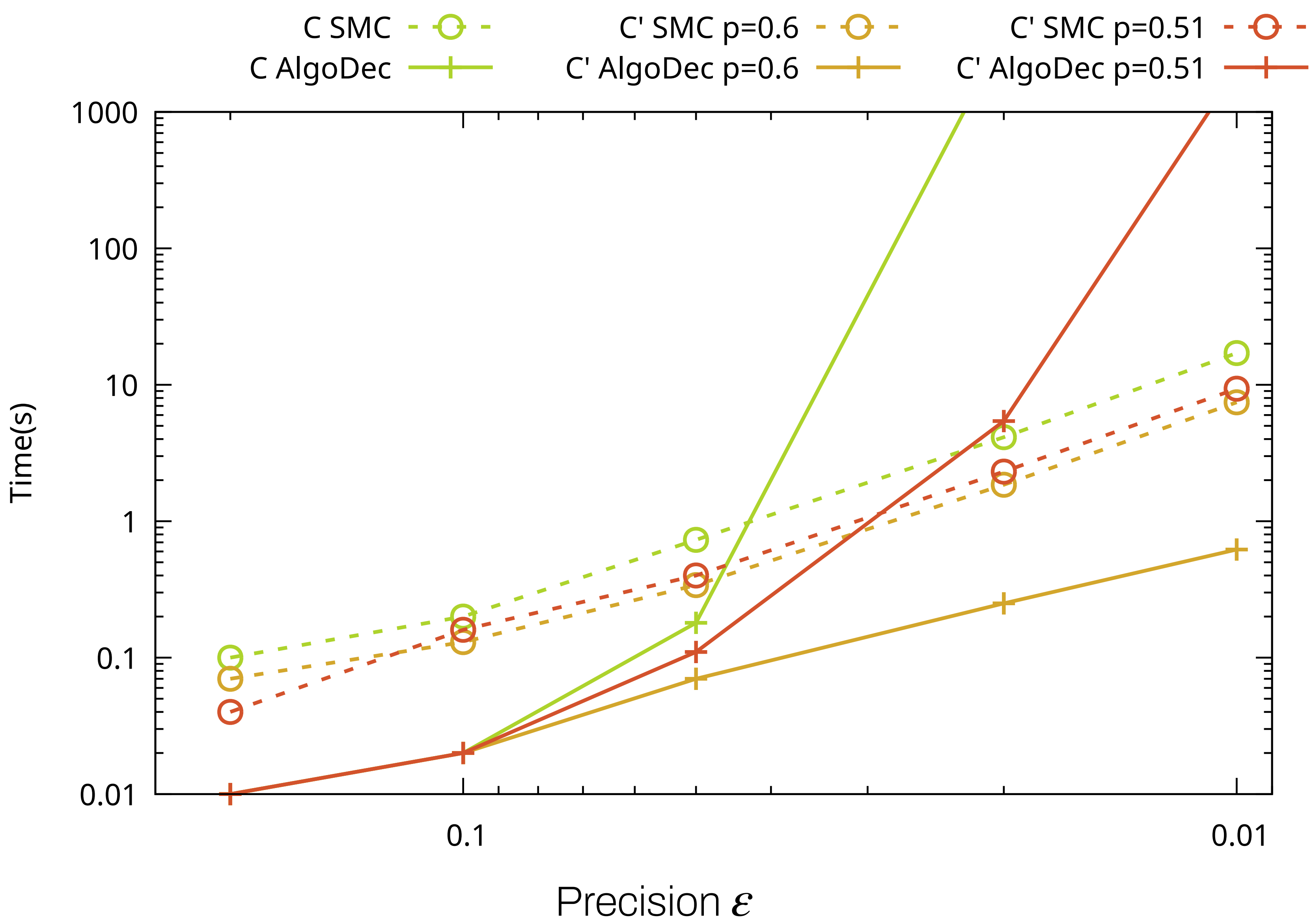

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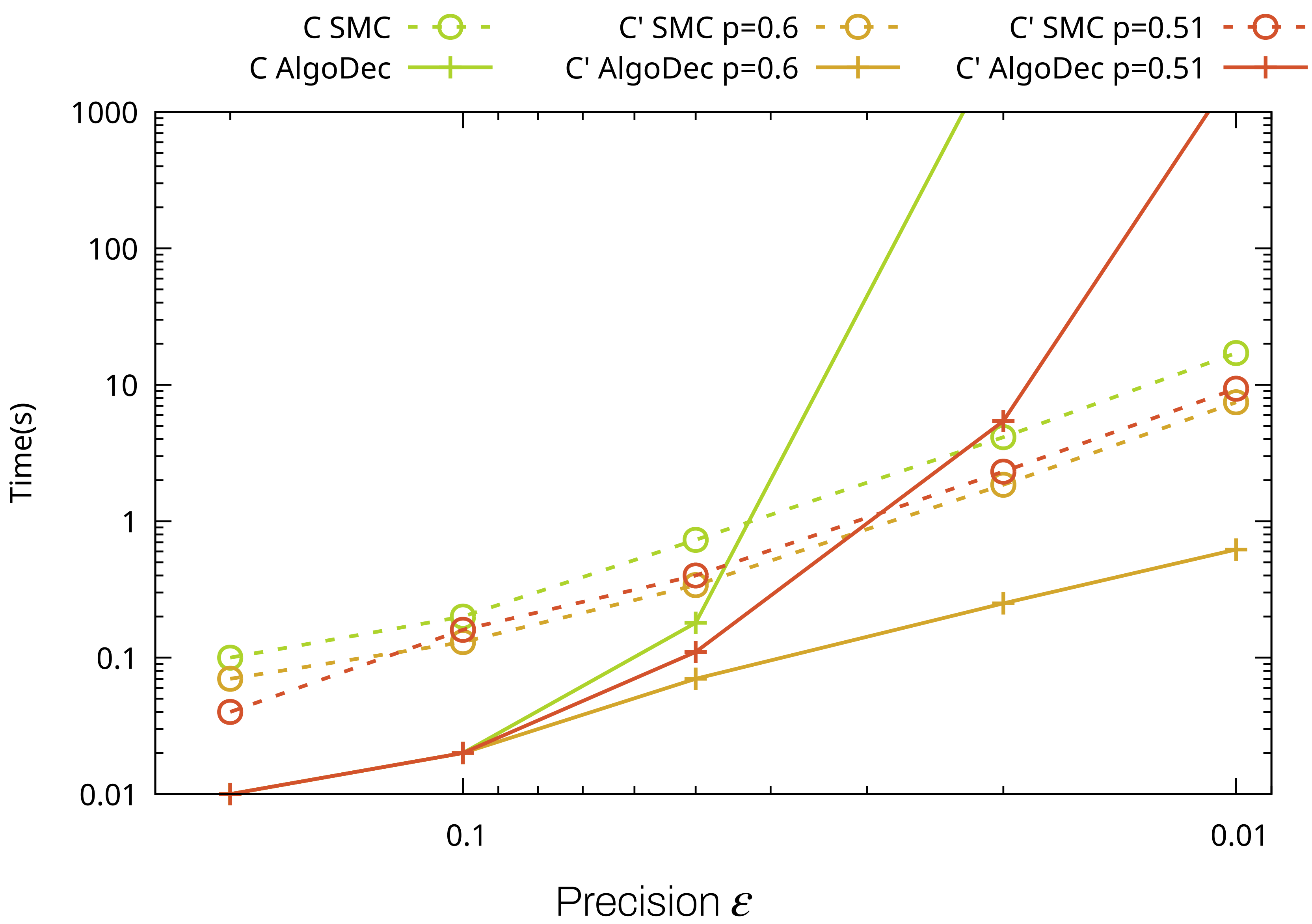

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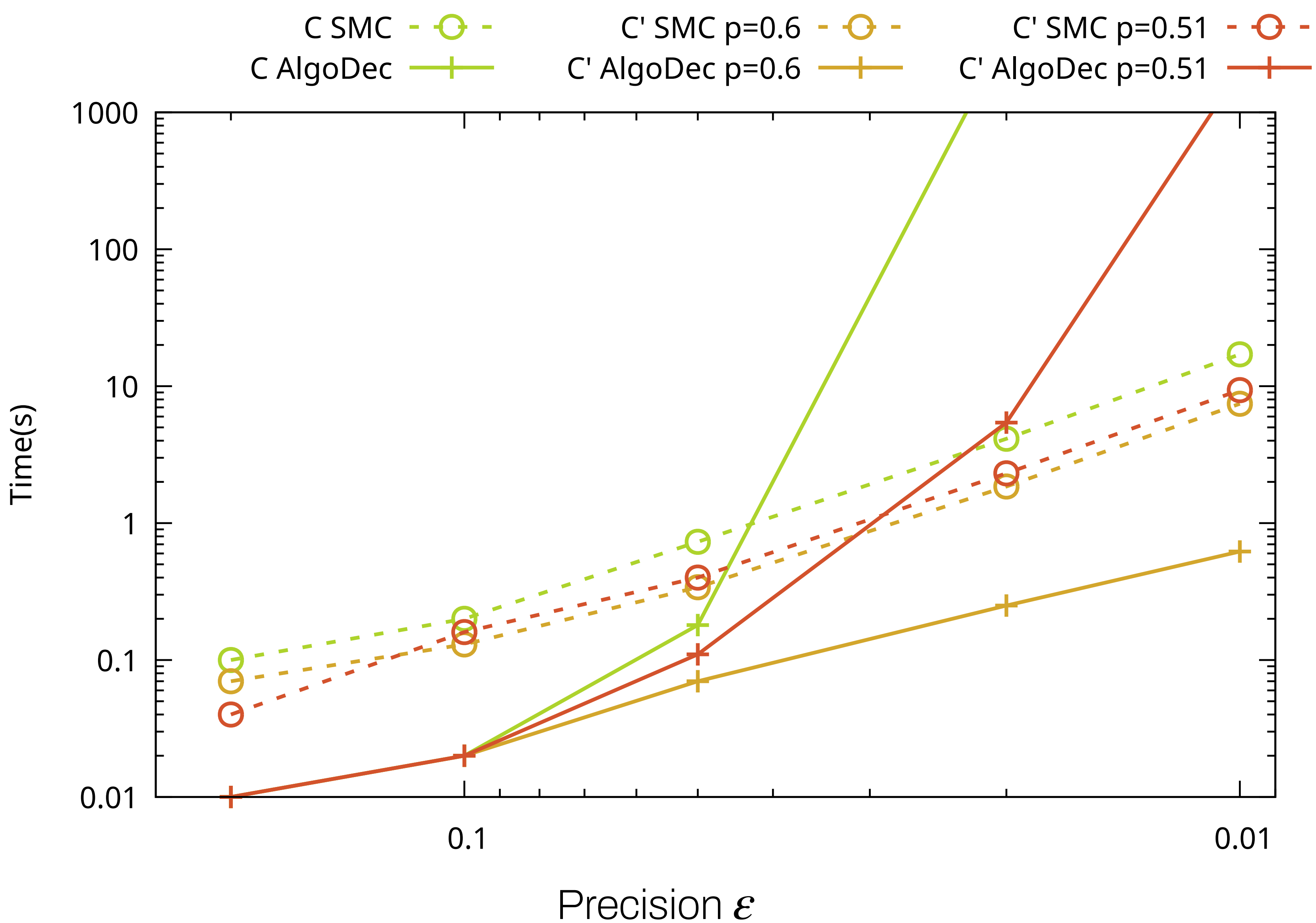

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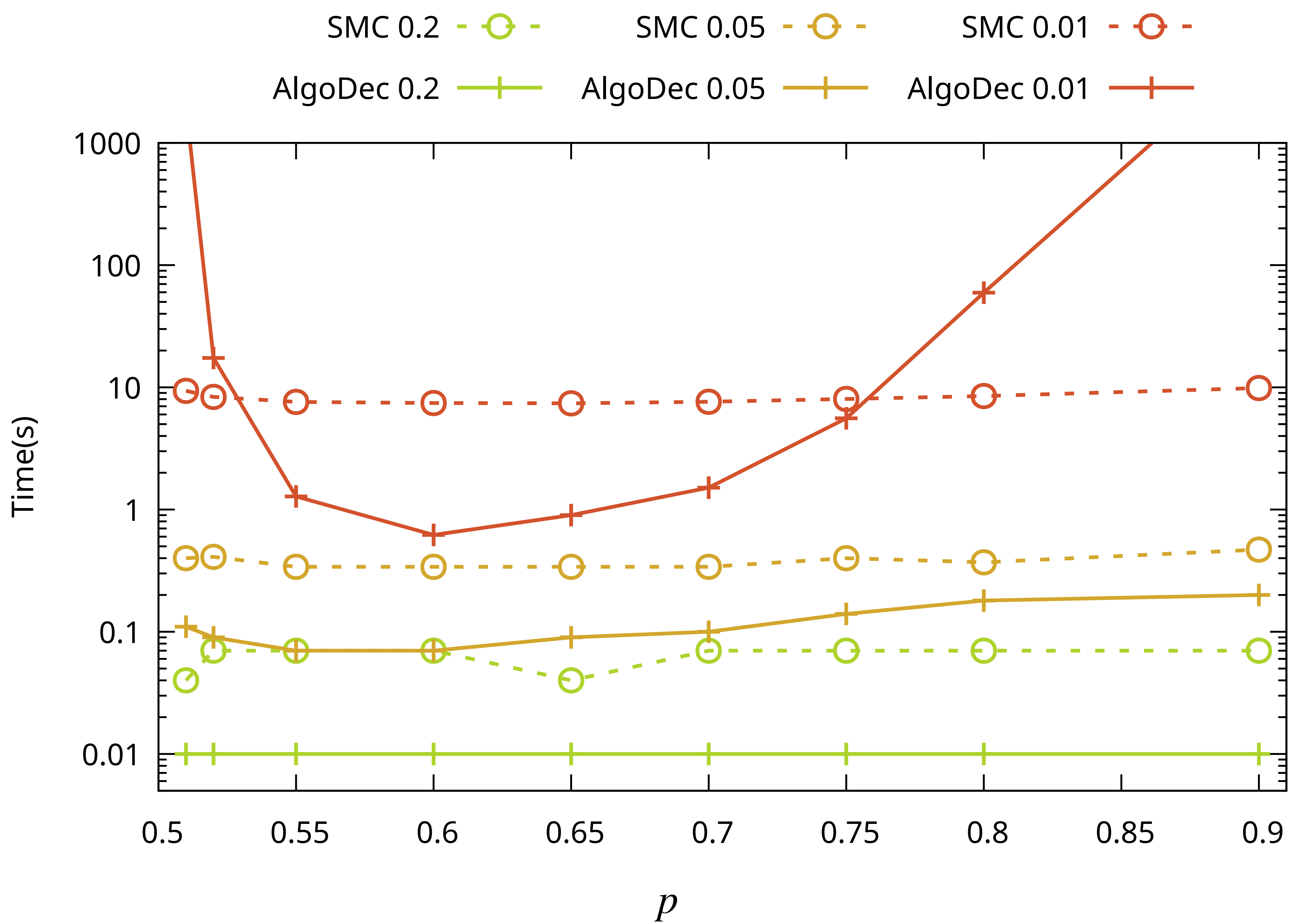
- For that best p, Approx behaves very well!

First example — continued



Second example

- State-free proba. pushdown automaton $\mathscr{C}$
  $A \xrightarrow{1} B \quad A \xrightarrow{1} C \quad B \xrightarrow{10} \varepsilon \quad B \xrightarrow{10+n} AA$
  $C \xrightarrow{10} A \quad C \xrightarrow{10+n} BB$
- Start from A, and target the empty stack

- It is not decisive
- It is p-divergent for every $1/2 < p < 1$

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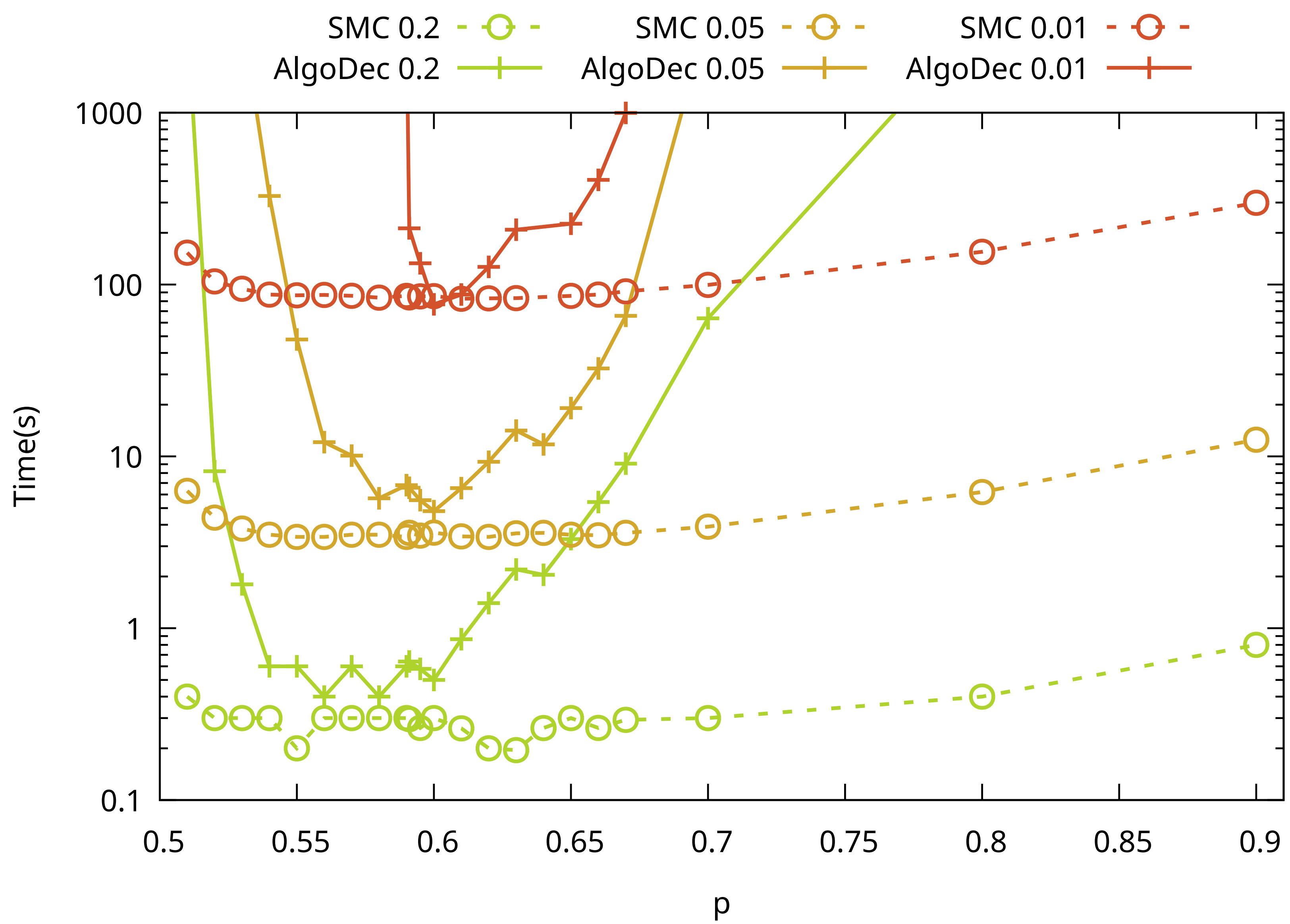

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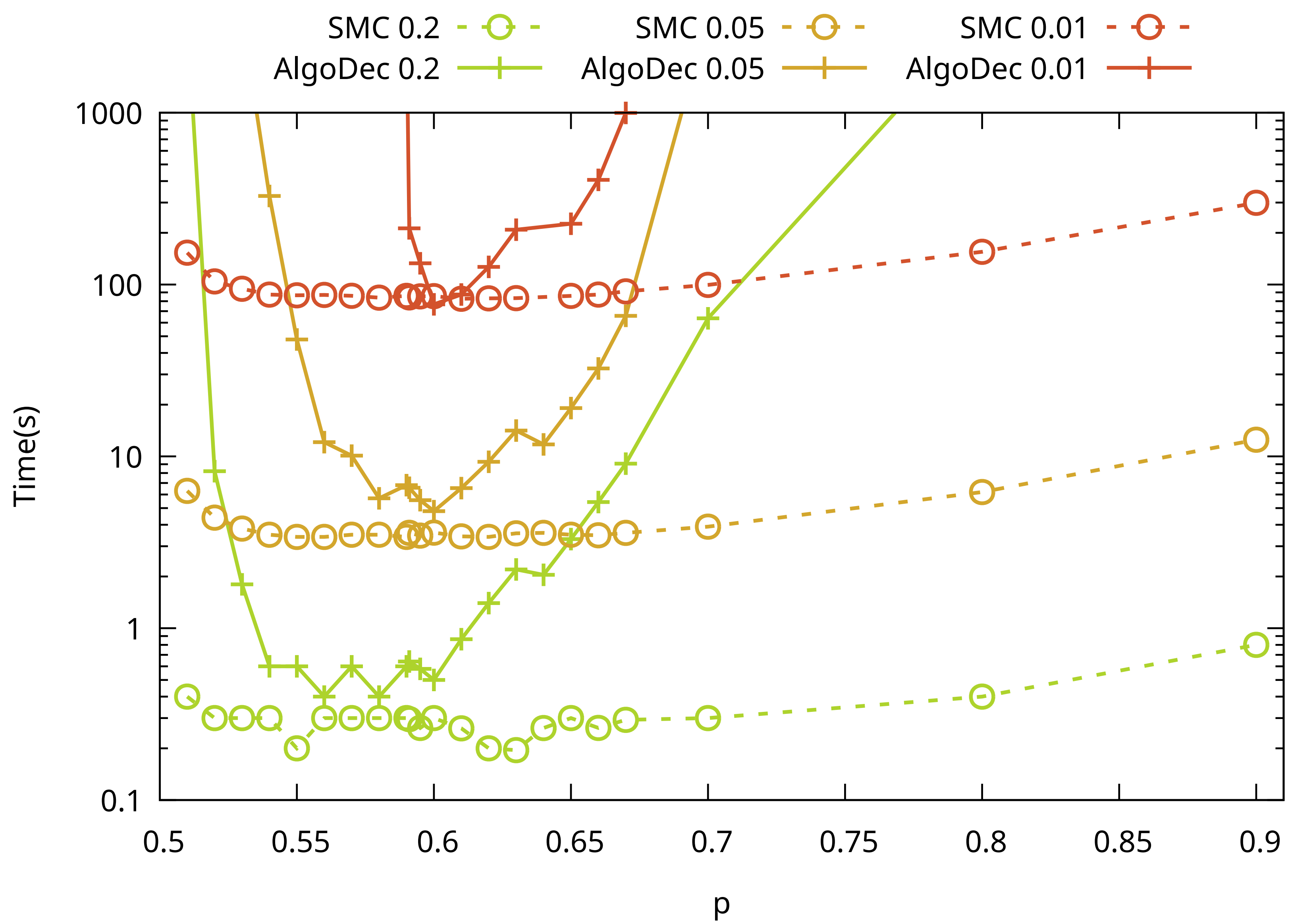

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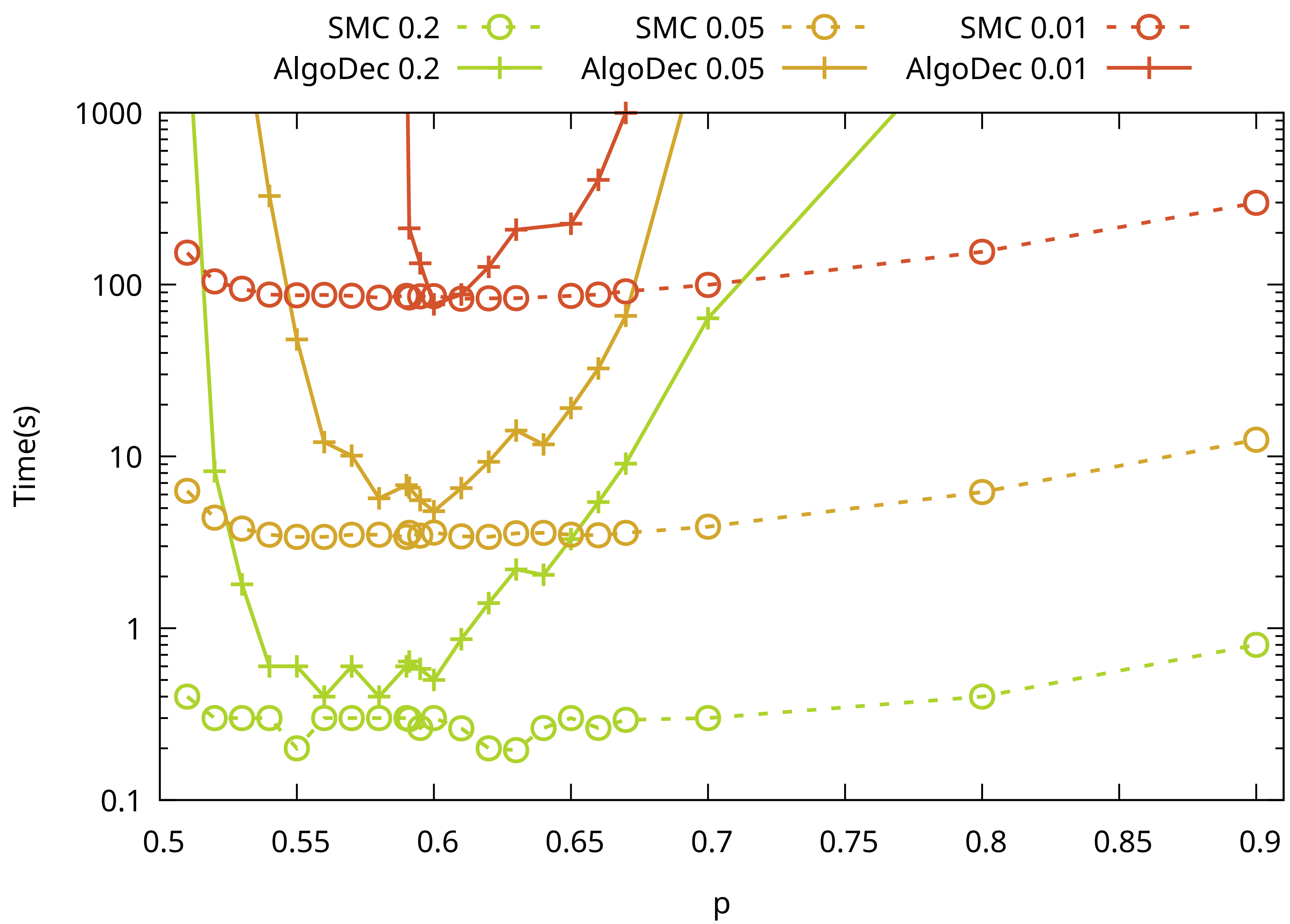

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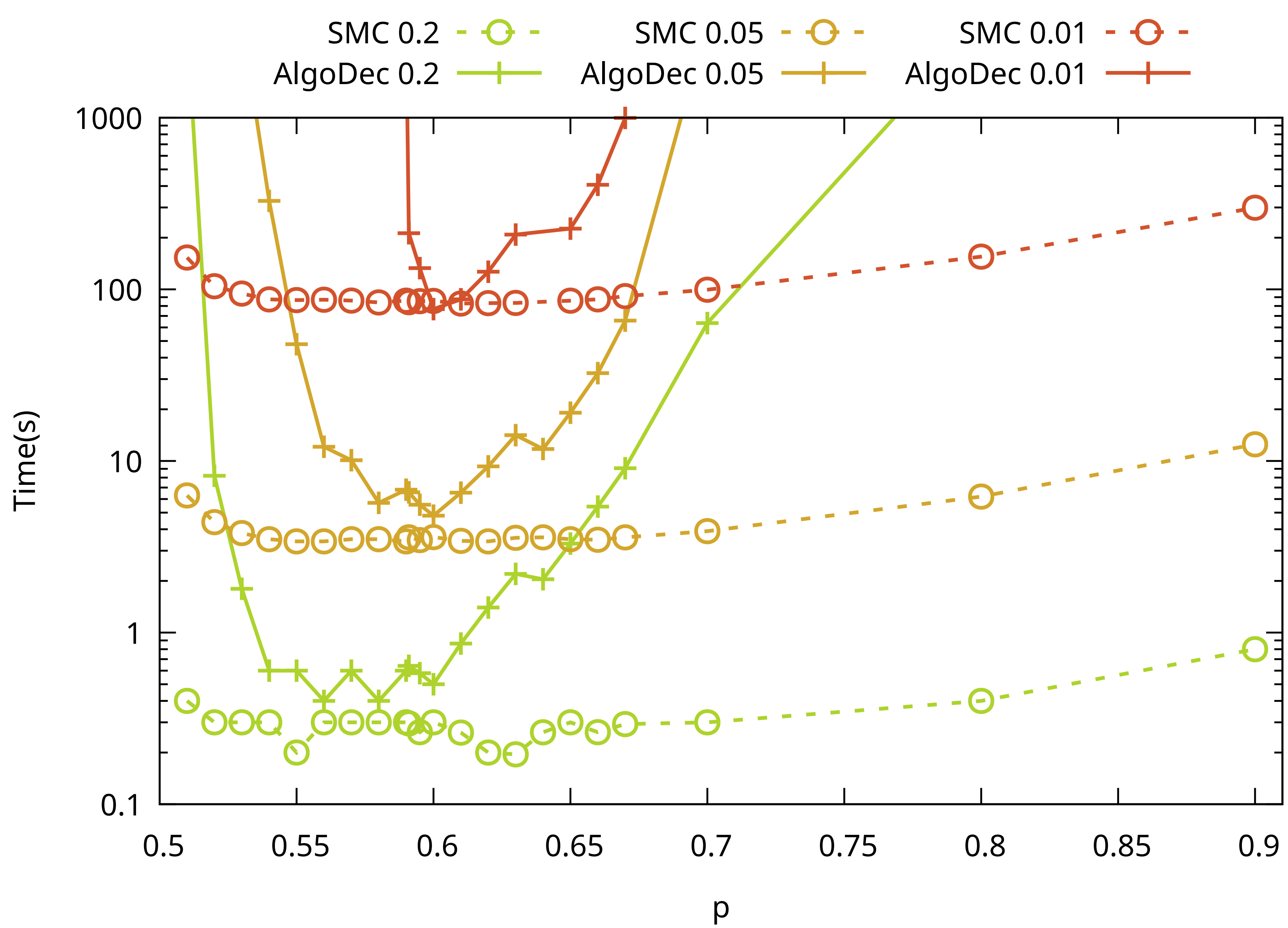

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- Approx very sensitive to p

Empirical conclusions

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- Suggests the following strategy:
  - Estimate the best p using Estim-SMC
  - Apply Approx on the corresponding biased Markov chain

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Some smoother conditions for application of the approach?

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